

Lösung A1

<p>a) $\frac{1}{\sqrt{5}} = \frac{1 \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{\sqrt{5}}{\sqrt{5}^2} = \frac{1}{5} \sqrt{5}$</p> <p>c) $\frac{2 \cdot \sqrt{26}}{\sqrt{26} \cdot \sqrt{26}} = \frac{2 \cdot \sqrt{26}}{\sqrt{26}^2} = \frac{2}{26} \sqrt{26} = \frac{1}{13} \sqrt{26}$</p> <p>e) $\frac{7 \cdot \sqrt{65}}{\sqrt{65} \cdot \sqrt{65}} = \frac{7 \cdot \sqrt{65}}{\sqrt{65}^2} = \frac{7}{65} \sqrt{65}$</p>	<p>b) $\frac{5 \cdot \sqrt{7}}{\sqrt{7} \cdot \sqrt{7}} = \frac{5 \cdot \sqrt{7}}{\sqrt{7}^2} = \frac{5}{7} \sqrt{7}$</p> <p>d) $\frac{5 \cdot \sqrt{11}}{\sqrt{11} \cdot \sqrt{11}} = \frac{5 \cdot \sqrt{11}}{\sqrt{11}^2} = \frac{5}{11} \sqrt{11}$</p> <p>f) $\frac{11 \cdot \sqrt{242}}{\sqrt{242} \cdot \sqrt{242}} = \frac{11 \cdot \sqrt{242}}{\sqrt{242}^2} = \frac{11 \cdot 11 \sqrt{2}}{242} = \frac{1}{2} \sqrt{2}$</p>
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Lösung A2

<p>a) $\sqrt{\frac{3}{5}} = \frac{\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{3} \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{\sqrt{15}}{\sqrt{5}^2} = \frac{1}{5} \sqrt{15}$</p> <p>c) $\sqrt{\frac{3}{13}} = \frac{\sqrt{3}}{\sqrt{13}} = \frac{\sqrt{3} \cdot \sqrt{13}}{\sqrt{13} \cdot \sqrt{13}} = \frac{1}{13} \sqrt{39}$</p> <p>e) $\sqrt{\frac{5}{17}} = \frac{\sqrt{5}}{\sqrt{17}} = \frac{\sqrt{5} \cdot \sqrt{17}}{\sqrt{17} \cdot \sqrt{17}} = \frac{1}{17} \sqrt{85}$</p>	<p>b) $\sqrt{\frac{7}{8}} = \frac{\sqrt{7}}{\sqrt{8}} = \frac{\sqrt{7} \cdot \sqrt{8}}{\sqrt{8} \cdot \sqrt{8}} = \frac{2 \cdot \sqrt{7} \cdot \sqrt{2}}{\sqrt{8}^2} = \frac{1}{4} \sqrt{14}$</p> <p>d) $\sqrt{\frac{8}{11}} = \frac{\sqrt{8}}{\sqrt{11}} = \frac{\sqrt{8} \cdot \sqrt{11}}{\sqrt{11} \cdot \sqrt{11}} = \frac{1}{11} \sqrt{22}$</p> <p>f) $\sqrt{\frac{6}{19}} = \frac{\sqrt{6}}{\sqrt{19}} = \frac{\sqrt{6} \cdot \sqrt{19}}{\sqrt{19} \cdot \sqrt{19}} = \frac{1}{19} \sqrt{113}$</p>
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Lösung A3

<p>a) $\frac{\sqrt{2} + \sqrt{3}}{\sqrt{3}} = \frac{(\sqrt{2} + \sqrt{3}) \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{6} + 3}{3}$</p> <p>c) $\frac{\sqrt{5} - \sqrt{2}}{\sqrt{5}} = \frac{(\sqrt{5} + \sqrt{2}) \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{5 + \sqrt{10}}{5}$</p> <p>e) $\frac{\sqrt{13} - \sqrt{7}}{2\sqrt{7}} = \frac{(\sqrt{13} - \sqrt{7}) \cdot \sqrt{7}}{2\sqrt{7} \cdot \sqrt{7}} = \frac{\sqrt{78} - 7}{14}$</p>	<p>b) $\frac{\sqrt{7} - \sqrt{12}}{\sqrt{7}} = \frac{(\sqrt{7} - \sqrt{12}) \cdot \sqrt{7}}{\sqrt{7} \cdot \sqrt{7}} = \frac{7 - 2\sqrt{21}}{7}$</p> <p>d) $\frac{\sqrt{6} + 2\sqrt{3}}{2\sqrt{3}} = \frac{(\sqrt{6} + 2\sqrt{3}) \cdot \sqrt{3}}{2\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{18} + 6}{6}$</p> <p>f) $\frac{\sqrt{15} + 3\sqrt{3}}{3\sqrt{3}} = \frac{(\sqrt{15} + 3\sqrt{3}) \cdot \sqrt{3}}{3\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{45} + 9}{9}$</p>
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Lösung A4

<p>a) $\frac{6}{\sqrt{3}} = \frac{6 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{6 \cdot \sqrt{3}}{3} = 2 \cdot \sqrt{3}$</p> <p>c) $\frac{6 \cdot \sqrt{5}}{5 \cdot \sqrt{2}} = \frac{6 \sqrt{5} \cdot \sqrt{2}}{5 \sqrt{2} \cdot \sqrt{2}} = \frac{6 \cdot \sqrt{10}}{10} = \frac{3}{5} \cdot \sqrt{10}$</p> <p>e) $\frac{\sqrt{3} + 2}{\sqrt{7}} = \frac{(\sqrt{3} + 2) \cdot \sqrt{7}}{\sqrt{7} \cdot \sqrt{7}} = \frac{\sqrt{21} + 2\sqrt{7}}{7}$</p> <p>g) $\sqrt{\frac{8}{3}} = \frac{\sqrt{8} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{24}}{3} = \frac{2}{3} \sqrt{6}$</p> <p>i) $\frac{x}{\sqrt{y+2}} = \frac{x \cdot \sqrt{y+2}}{\sqrt{y+2} \cdot \sqrt{y+2}} = \frac{x \cdot \sqrt{y+2}}{y+2}$</p>	<p>b) $\frac{\sqrt{5}}{3\sqrt{2} - \sqrt{3}} = \frac{\sqrt{5} \cdot (3\sqrt{2} + \sqrt{3})}{(3\sqrt{2} - \sqrt{3}) \cdot (3\sqrt{2} + \sqrt{3})} = \frac{3\sqrt{10} + \sqrt{15}}{15}$</p> <p>d) $\frac{2\sqrt{7}}{3\sqrt{7} + 7} = \frac{2\sqrt{7} \cdot (3\sqrt{7} - 7)}{(3\sqrt{7} + 7) \cdot (3\sqrt{7} - 7)} = \frac{42 - 14\sqrt{7}}{14} = 3 - \sqrt{7}$</p> <p>f) $\frac{2\sqrt{5} - 3\sqrt{2}}{2\sqrt{5} + 3\sqrt{2}} = \frac{(2\sqrt{5} - 3\sqrt{2}) \cdot (2\sqrt{5} - 3\sqrt{2})}{(2\sqrt{5} + 3\sqrt{2}) \cdot (2\sqrt{5} - 3\sqrt{2})} = \frac{20 - 12\sqrt{10} + 18}{20 - 18}$ $= \frac{2 - 12\sqrt{10}}{2} = 1 - 6\sqrt{10}$</p> <p>h) $\frac{\sqrt{a}}{\sqrt{a} + \sqrt{b}} = \frac{\sqrt{a} \cdot (\sqrt{a} - \sqrt{b})}{(\sqrt{a} + \sqrt{b}) \cdot (\sqrt{a} - \sqrt{b})} = \frac{a - \sqrt{ab}}{a - b}$</p> <p>j) $\frac{4\sqrt{2} - 6\sqrt{5}}{3\sqrt{2} - 2\sqrt{5}} = \frac{(4\sqrt{2} - 6\sqrt{5}) \cdot (3\sqrt{2} + 2\sqrt{5})}{(3\sqrt{2} - 2\sqrt{5}) \cdot (3\sqrt{2} + 2\sqrt{5})} = \frac{24 + 8\sqrt{10} - 18\sqrt{10} - 60}{18 - 20}$ $= \frac{-36 - 8\sqrt{10}}{-2} = 18 + 4\sqrt{10}$</p>
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Lösung A5

a) $\frac{1}{\sqrt{x}} = \frac{\sqrt{x}}{\sqrt{x} \cdot \sqrt{x}} = \frac{\sqrt{x}}{x} = \frac{1}{x} \cdot \sqrt{x}$
 $x > 0$

c) $\frac{3}{3-\sqrt{x}} = \frac{3 \cdot (3+\sqrt{x})}{(3-\sqrt{x}) \cdot (3+\sqrt{x})} = \frac{6+3\sqrt{x}}{9-x}$
 $x \geq 0 \wedge x \neq 9$

e) $\frac{\sqrt{a}}{\sqrt{a}+\sqrt{b}} = \frac{\sqrt{a} \cdot (\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b}) \cdot (\sqrt{a}-\sqrt{b})} = \frac{a-\sqrt{ab}}{a-b}$
 $a; b \geq 0 \wedge a - b \neq 0$

g) $\frac{2}{\sqrt{a-b}} = \frac{2 \cdot \sqrt{a-b}}{\sqrt{a-b}^2} = \frac{2 \cdot \sqrt{a-b}}{a-b}$
 $a \geq b \wedge a \neq b \Rightarrow a > b$

i) $\frac{3\sqrt{a}}{\sqrt{b}-\sqrt{a}} = \frac{3\sqrt{a} \cdot (\sqrt{b}+\sqrt{a})}{(\sqrt{b}-\sqrt{a}) \cdot (\sqrt{b}+\sqrt{a})} = \frac{a(\sqrt{ab}+a)}{b-a}$
 $a; b \geq 0 \wedge b - a \neq 0$

k) $\frac{\sqrt{5a}}{5-\sqrt{5a}} = \frac{\sqrt{5a} \cdot (5+\sqrt{5a})}{(5-\sqrt{5a}) \cdot (5+\sqrt{5a})} = \frac{5(\sqrt{5a}+5a)}{25-a}$
 $a \geq 0 \wedge a \neq 25$

b) $\frac{\sqrt{a}}{\sqrt{a+b}} = \frac{\sqrt{a} \cdot \sqrt{a+b}}{\sqrt{a+b} \cdot \sqrt{a+b}} = \frac{\sqrt{a} \cdot \sqrt{a+b}}{a+b}$
 $a \geq 0 \wedge a > b$

d) $\frac{2}{3\sqrt{a}} = \frac{2 \cdot \sqrt{a}}{3 \cdot \sqrt{a} \cdot \sqrt{a}} = \frac{2\sqrt{a}}{3a}$
 $a > 0$

f) $\frac{b}{\sqrt{b}-2} = \frac{b \cdot (\sqrt{b}+2)}{(\sqrt{b}-2) \cdot (\sqrt{b}+2)} = \frac{b \cdot (\sqrt{b}+2)}{b-4}$
 $b \geq 0$

h) $\frac{\sqrt{a}}{4+\sqrt{a}} = \frac{\sqrt{a} \cdot (4-\sqrt{a})}{(4+\sqrt{a}) \cdot (4-\sqrt{a})} = \frac{4\sqrt{a}-a}{16-a}$
 $a \geq 0 \wedge a \neq 16$

j) $\frac{\sqrt{x-y}}{\sqrt{x+y}} = \frac{\sqrt{x-y} \cdot \sqrt{x+y}}{\sqrt{x+y}^2} = \frac{\sqrt{x^2-y^2}}{x+y}$
 $x > -y \wedge x - y \geq 0 \Rightarrow x \geq y$

l) $\frac{2}{2+\sqrt{x}} = \frac{2 \cdot (2-\sqrt{x})}{(2+\sqrt{x}) \cdot (2-\sqrt{x})} = \frac{4-2\sqrt{x}}{4-x}$
 $x \geq 0 \wedge x \neq 4$