



Aufgabe A1

Bestimme die 1. und 2. Ableitung der folgenden Funktionsgleichungen:

a) $f(x) = (5^x - 2^x)^3$

b) $f(x) = \left(\frac{1}{2}\right)^x \cdot \left(3x + \frac{1}{x}\right)$

c) $f(x) = (2^{3x} - x) \cdot x^4$

d) $f(x) = 3^x \cdot x^3$

e) $f(x) = \sin\left(\left(\frac{3}{2}\right)^x - 5\right)$

f) $f(x) = e^x \cdot x^e$

g) $f(x) = x^2 \cdot 2^{-x}$

h) $f(x) = x \cdot 2^{\sin(2^x)}$

i) $f_a(x) = \frac{1}{a^{0,5x}} + ax$

j) $f_a(x) = \frac{a^{\sqrt{x}}}{x-1}$

k) $f(t) = t^2 \cdot a^t$

l) $f_a(t) = \frac{a^t}{t} \cdot a^{-2t}$

m) $f_k(t) = k^2 t \cdot \sqrt[3]{15k^t}$

Aufgabe A2

Zeige, dass die Ableitung der Funktion f mit $f(x) = x^{e^x}$ die Funktion f' mit $f'(x) = x^{e^x} \cdot e^x \cdot \left(\frac{1}{x} + \ln(x)\right)$ ist.

Lösung A1

a) $f(x) = (5^x - 2^x)^3$

$$f'(x) = 3 \cdot (5^x - 2^x)^2 \cdot (\ln(5) \cdot 5^x + \ln(3) \cdot 3^x)$$

$$u = 3 \cdot (5^x - 2^x)^2 \quad u' = 6 \cdot (5^x - 2^x) \cdot (\ln(5) \cdot 5^x - \ln(2) \cdot 2^x)$$

$$v = \ln(5) \cdot 5^x - \ln(2) \cdot 2^x \quad v' = \ln^2(5) \cdot 5^x - \ln^2(2) \cdot 2^x$$

$$\begin{aligned} f''(x) &= 6 \cdot (5^x - 2^x) \cdot (\ln(5) \cdot 5^x - \ln(2) \cdot 2^x) \cdot (\ln(5) \cdot 5^x - \ln(2) \cdot 2^x) + \\ &\quad 3 \cdot (5^x - 2^x)^2 \cdot (\ln^2(5) \cdot 5^x - \ln^2(2) \cdot 2^x) \\ &= 6 \cdot (5^x - 2^x) \cdot (\ln(5) \cdot 5^x - \ln(2) \cdot 2^x)^2 + \\ &\quad 3 \cdot (5^x - 2^x)^2 \cdot (\ln^2(5) \cdot 5^x - \ln^2(2) \cdot 2^x) \end{aligned}$$

b) $f(x) = \left(\frac{1}{2}\right)^x \cdot \left(3x + \frac{1}{x}\right)$

$$u = \frac{1}{2^x} \quad u' = \frac{-\ln(2)}{2^x}$$

$$v = 3x + \frac{1}{x} \quad v' = 3 - \frac{1}{x^2}$$

$$f'(x) = \frac{-\ln(2) \cdot (3x + \frac{1}{x})}{2^x} + \frac{3 - \frac{1}{x^2}}{2^x} = \frac{-\ln(2) \cdot (3x + \frac{1}{x}) + 3 - \frac{1}{x^2}}{2^x}$$

$$u = -\ln(2) \cdot \left(3x + \frac{1}{x}\right) + 3 - \frac{1}{x^2}$$

$$u' = -\ln(2) \cdot \left(3 - \frac{1}{x^2}\right) + \frac{2}{x^3}$$

$$v = 2^x \quad v' = \ln(2) \cdot 2^x$$

$$\begin{aligned} f''(x) &= \frac{2^x \cdot (-\ln(2) \cdot (3 - \frac{1}{x^2}) + \frac{2}{x^3}) - 2^x \cdot \ln(2) \cdot (-\ln(2) \cdot (3x + \frac{1}{x}) + 3 - \frac{1}{x^2})}{2^{2x}} \\ &= \frac{-\ln(2) \cdot (3 - \frac{1}{x^2}) + \frac{2}{x^3} - (\ln(2) \cdot (-\ln(2) \cdot (3x + \frac{1}{x}) + 3 - \frac{1}{x^2}))}{2^x} \\ &= \frac{-\ln(2) \cdot 3 + \frac{\ln(2)}{x^2} + \frac{2}{x^3} + \ln^2(2) \cdot (3x + \frac{1}{x}) - \ln(2) \cdot 3 + \frac{\ln(2)}{x^2}}{2^x} = \frac{-6 \cdot \ln(2) + \frac{2 \cdot \ln(2)}{x^2} + \frac{2}{x^3} + 3x \cdot \ln^2(2) + \frac{\ln^2(2)}{x}}{2^x} \\ &= \frac{-6x^3 \cdot \ln(2) + 2x \cdot \ln(2) + 2 + 3x^4 \cdot \ln^2(2) + x^2 \cdot \ln^2(2)}{x^3 \cdot 2^x} \end{aligned}$$

c) $f(x) = (2^{3x} - x) \cdot x^4$

$$u = 2^{3x} - x \quad u' = 3 \ln(2) \cdot 2^{3x} - 1$$

$$v = x^4 \quad v' = 4x^3$$

$$\begin{aligned} f'(x) &= (3 \ln(2) \cdot 2^{3x} - 1) \cdot x^4 + 4x^3(2^{3x} - x) \\ &= x^3 \cdot ((3 \ln(2) x + 4) \cdot 2^{3x} - 5x) \end{aligned}$$

$$\begin{aligned} f''(x) &= x^3 \cdot (3 \ln(2) (3 \ln(2) x + 4) \cdot 2^{3x} + 3 \ln(2) \cdot 2^{3x} - 5) + \\ &\quad 3x^2((3 \ln(2) x + 4)) \end{aligned}$$

$$f''(x) = x^2((9 \ln^2(2)x^2 + 24 \ln(2)x + 12) \cdot 2^{3x} - 20x)$$

d) $f(x) = 3^x \cdot x^3$

$$u = 3^x \quad u' = \ln(3) \cdot 3^x$$

$$v = x^3 \quad v' = 3x^2$$

$$f'(x) = \ln(3) \cdot 3^x \cdot x^3 + 3x^2 \cdot 3^x = x^2 \cdot 3^x \cdot (3 + \ln(3) \cdot x)$$

$$f''(x) = x \cdot 3^x (\ln^2(3) \cdot x^2 + 6 \ln(3)x + 6)$$

e) $f(x) = \sin\left(\left(\frac{3}{2}\right)^x - 5\right)$

$$f'(x) = \cos\left(\left(\frac{3}{2}\right)^x - 5\right) \cdot (\ln(3) - \ln(2)) \cdot \left(\frac{3}{2}\right)^x$$

$$u = (\ln(3) - \ln(2)) \cdot \cos\left(\left(\frac{3}{2}\right)^x - 5\right)$$

$$u' = -(\ln(3) - \ln(2))^2 \cdot \sin\left(\left(\frac{3}{2}\right)^x - 5\right) \cdot \left(\frac{3}{2}\right)^x$$

$$v = \left(\frac{3}{2}\right)^x \quad v' = (\ln(3) - \ln(2)) \cdot \left(\frac{3}{2}\right)^x$$

$$f''(x) = -(\ln(3) - \ln(2))^2 \cdot \sin\left(\left(\frac{3}{2}\right)^x - 5\right) \cdot \left(\frac{3}{2}\right)^{2x} + (\ln(3) - \ln(2))^2 \cdot \cos\left(\left(\frac{3}{2}\right)^x - 5\right) \cdot \left(\frac{3}{2}\right)^x$$

$$f''(x) = (\ln(3) - \ln(2))^2 \cdot \left(\frac{3}{2}\right)^x \cdot \left(\cos\left(\left(\frac{3}{2}\right)^x - 5\right) - \left(\frac{3}{2}\right)^x \cdot \sin\left(\left(\frac{3}{2}\right)^x - 5\right)\right)$$

f) $f(x) = e^x \cdot x^e$ $u = e^x$ $u' = e^x$
 $v = x^e$ $v' = e \cdot x^{e-1}$

$$f'(x) = e^x \cdot x^e + e^{x+1} \cdot x^{e-1}$$

$$s = e^{x+1}$$

$$t = x^{e-1}$$

$$s' = e^{x+1}$$

$$v' = (e-1) \cdot x^{e-2}$$

$$f''(x) = f'(x) + e^{x+1} \cdot x^{e-1} + e^{x+1} \cdot (e-1) \cdot x^{e-2} = e^x \cdot x^e + 2e^{x+1} \cdot x^{e-1} + e^{x+1} \cdot (e-1) \cdot x^{e-2}$$

g) $f(x) = x^2 \cdot 2^{-x}$ $u = x^2$ $u' = 2x$
 $v = \frac{1}{2^x}$ $v' = -\frac{\ln(2)}{2^x}$

$$f'(x) = \frac{2x - \ln(2) \cdot x^2}{2^x}$$

$$u = 2x - \ln(2) \cdot x^2$$

$$v = 2^x$$

$$u' = 2 - 2\ln(2) \cdot x$$

$$v' = \ln(2) \cdot 2^x$$

$$f''(x) = \frac{(2-2\ln(2) \cdot x) \cdot 2^x - \ln(2) \cdot 2^x \cdot (2x - \ln(2) \cdot x^2)}{2^{2x}} = \frac{\ln^2(2) \cdot x^2 - 4\ln(2) \cdot x + 2}{2^x}$$

h) $f(x) = x \cdot 2^{\sin(2x)}$ $u = x$ $u' = 1$
 $v = 2^{\sin(2x)}$ $v' = \ln^2(2) \cdot 2^{\sin(2x)+x} \cdot \cos(2x)$

$$f'(x) = 2^{\sin(2x)} + x \cdot \ln^2(2) \cdot 2^{\sin(2x)+x} \cdot \cos(2x) = 2^{\sin(2x)} \cdot (1 + \ln^2(2) \cdot x \cdot 2^x \cdot \cos(2x))$$

$$f''(x) = -\ln^2(2) \cdot 2^{\sin(2x)+x} \cdot (\ln(2) \cdot x \cdot 2^x \sin(2x) - \ln^2(2) \cdot x \cdot 2^x \cos(2x)) + (-2\ln(2)x - 2) \cos(2x)$$

i) $f_a(x) = \frac{1}{a^{0,5x}} + ax$

$$f_a'(x) = a - \frac{\ln(a)}{2a^{0,5x}}$$

$$f_a''(x) = \frac{\ln^2(a)}{4a^{0,5x}}$$

j) $f_a(x) = \frac{a^{\sqrt{x}}}{x-1}$

$$f_a'(x) = \frac{\ln(a) \cdot a^{\sqrt{x}}}{2(x-1)\sqrt{x}} - \frac{a^{\sqrt{x}}}{(x-1)^2}$$

$$f_a''(x) = -\frac{\ln(a) \cdot a^{\sqrt{x}}}{(x-1)^2 \sqrt{x}} + \frac{\ln^2(a) \cdot a^{\sqrt{x}}}{4x(x-1)} - \frac{\ln(a) \cdot a^{\sqrt{x}}}{4x\sqrt{x}(x-1)} + \frac{2 \cdot a^{\sqrt{x}}}{(x-1)^3}$$

k) $f(t) = t^2 \cdot a^t$ $u = t^2$ $u' = 2t$
 $v = a^t$ $v' = \ln(a) \cdot a^t$

$$f'(t) = 2t \cdot a^t + \ln(a) \cdot t^2 \cdot a^t$$

$$f''(t) = a^t \cdot (\ln^2(a) \cdot t^2 + 4\ln(a) \cdot t + 2)$$

l) $f_a(t) = \frac{a^t}{t} \cdot a^{-2t}$

$$f_a'(t) = -\frac{\ln(a)t+1}{a^t t^2}$$

$$f_a''(t) = \frac{\ln^2(a)t^2 + 2\ln(a)t + 2}{a^t t^3}$$

m) $f_k(t) = k^2 t \cdot \sqrt[3]{15k^t}$

$$f'(x) = \frac{\sqrt[3]{15k^t} \cdot (\ln(k)t + 3)}{3}$$

$$f''(x) = \frac{\sqrt[3]{15k} \frac{7+x}{3} \cdot \ln(k) \cdot (\ln(k)t + 6)}{9}$$

Lösung A2

$$f(x) = x e^x = e^{\ln(x) \cdot e^x} = e^{u(x)} \text{ mit}$$

$$u(x) = \ln(x) \cdot e^x$$

$$v = \ln(x)$$

$$v' = \frac{1}{x}$$

$$w = e^x$$

$$w' = e^x$$

$$u'(x) = \frac{e^x}{x} + \ln(x) \cdot e^x$$

$$f(x) = e^{u(x)}$$

$$f'(x) = u'(x) \cdot e^{u(x)}$$

$$f'(x) = \left(\frac{e^x}{x} + \ln(x) \cdot e^x \right) \cdot e^{u(x)} = e^x \left(\frac{1}{x} + \ln(x) \right) \cdot e^{u(x)}$$

$$f'(x) = x e^x \cdot e^x \cdot \left(\frac{1}{x} + \ln(x) \right)$$

q.e.d.