Lösung A1

a) Produktregel: $f_1(x) = 3x^2 \cdot (x+3)^{-1}$

$f_1{}'(x) = 6x \cdot (x+3)^{-1} - 3x^2 \cdot (x+3)^{-2}$

Quotientenregel: $f_1(x) = \frac{3x^2}{x+3}$

$f_1{}'(x) = \frac{6x\cdot(x+3)-3x^2}{(x+3)^2} = \frac{3x^2+18x}{(x+3)^2}$

b) Produktregel: $f_2(x) = (2x+3) \cdot x^{-2}$

$f_2{}'(x) = 2 \cdot x^{-2} - 2 \cdot (2x+3) \cdot x^{-3}$

Quotientenregel: $f_2(x) = \frac{2x+3}{x^2}$

$f_2{}'(x) = \frac{2x^2-2x(2x+3)}{x^4} = -\frac{2x^2+6x}{x^4} = -\frac{2x+6}{x^3}$

c) Produktregel: $f_3(x) = 4x \cdot (x^3+3x^2)^{-1}$

$f_3{}'(x) = 4 \cdot (x^3+3x^2)^{-1} + 4x \cdot (x^3+3x^2)^{-2}$

Quotientenregel: $f_3(x) = \frac{4x}{x^3+3x^2}$

$f_3{}'(x) = \frac{4\cdot(x^3+3x^2)-4x(3x^2+6x)}{x^4\cdot(x+3)^2} = \frac{4x^3+12x^2-12x^3-24x^2}{x^4\cdot(x+3)^2} = \frac{-8x^3-12x^2}{x^4(x+3)^2} = -\frac{12+8x}{x^2\cdot(x+3)^2}$

d) Produktregel: $f_4(x) = (x^3-4x) \cdot (7x^2)^{-1}$

$f_4{}'(x) = (3x^2-4) \cdot (7x^2)^{-1} + 14x \cdot (x^3-4x) \cdot (7x^2)^{-2}$

Quotientenregel: $f_4(x) = \frac{x^3-4x}{7x^2}$

$f_4{}'(x) = \frac{(3x^2-4)\cdot 7x^2-14x(x^3-4x)}{49x^4} = \frac{21x^4-28x^2-14x^4+56x^2}{49x^4} = \frac{7x^2+28}{49x^2} = \frac{x^2+4}{7x^2}$

e) Produktregel: $f_5(x) = (3x^2-x^5) \cdot (x+1)^{-1}$

$f_5{}'(x) = (6x-5x^4) \cdot (x+1)^{-1} + (3x^2-x^5) \cdot (x+1)^{-2}$

Quotientenregel: $f_5(x) = \frac{3x^2-x^5}{x+1}$

$f_5{}'(x) = \frac{(6x-5x^4)\cdot(x+1)-(3x^2-x^5)}{(x+1)^2} = \frac{6x^2+6x-5x^5-5x^4-3x^2+x^5}{(x+1)^2} = \frac{-4x^5-5x^4+3x^2+6x}{(x+1)^2}$

f) Produktregel: $f_6(x) = (x^2+x) \cdot (7x^3-x)^{-1}$

$f_6{}'(x) = (2x+1) \cdot (7x^3-x)^{-1} + (21x^2-1) \cdot (x^2+x) \cdot (7x^3-x)^{-2}$

Quotientenregel: $f_6(x) = \frac{x^2+x}{7x^3-x}$

$f_6{}'(x) = \frac{(2x+1)\cdot(7x^3-x)-(21x^2-1)\cdot(x^2+x)}{x^2\cdot(7x^2-1)^2} = \frac{14x^4-2x^2+7x^3-x-(21x^4+21x^3-x^2-x)}{x^2\cdot(7x^2-1)^2}$

$= \frac{-7x^4-14x^3-x^2}{x^2\cdot(7x^2-1)^2} = -\frac{7x^2+14x+1}{(7x^2-1)^2}$

Lösung A2

a) $f_1(x) = \frac{x}{x+3}$; $\mathbb{D} = x \in \mathbb{R}\backslash\{-3\}$

$f_1{}'(x) = \frac{x+3-x}{(x+3)^2} = \frac{3}{(x+3)^2}$

b) $f_2(x) = \frac{x-3}{x^2}$; $\mathbb{D} = x \in \mathbb{R}\backslash\{0\}$

$f_2{}'(x) = \frac{x^2-2x(x-3)}{x^4} = \frac{-x^2+6x}{x^4} = -\frac{x-6}{x^3}$

c) $f_3(x) = \frac{3x^3+x^2}{(5x)^5}$; $\mathbb{D} = x \in \mathbb{R}\backslash\{0\}$

$$f_3{}'(x) = \frac{(9x^2+2x)\cdot(5x)^5-25(3x^3+x^2)\cdot(5x)^4}{(5x)^{10}} = \frac{(5x)^4\cdot\left((9x^2+2\cdot x)\cdot 5x-25\cdot(3x^3+x^2)\right)}{(5x)^{10}} = -\frac{5x^2(6x+3)}{(5x)^6}$$
$$= -\frac{6x+3}{625x^4}$$

d) $f_4(x) = \frac{3x^2-x}{8x^3}$; $\mathbb{D} = x \in \mathbb{R}\backslash\{0\}$

$$f_4{}'(x) = \frac{(6x-1)8x^3-24x^2(3x^2-x)}{64x^6} = \frac{48x^4-8x^3-72x^4+24x^3}{64x^6} = \frac{8x^3(-3x+2)}{64x^6} = -\frac{3x-2}{8x^3}$$

e) $f_5(x) = \frac{2x^3}{(x^5-x^4)^3}$·; $\mathbb{D} = x \in \mathbb{R}\backslash\{0;1\}$

$$f_5{}'(x) = \frac{6x^2\cdot(x^5-x^4)^3-6x^3\cdot(x^5-x^4)^2\cdot(5x^4-4x^3)}{(x^5-x^4)^6} = \frac{(x^5-x^4)^2\cdot\left(6x^2\cdot(x^5-x^4)-6x^3\cdot(5x^4-4x^3)\right)}{(x^5-x^4)^6}$$
$$= \frac{6x^7-6x^6-30x^7+24x^6}{(x^5-x^4)^4} = \frac{-24x^7+18x^6}{x^{16}(x-1)^4} = -\frac{24x-18}{x^{10}\cdot(x-1)^4}$$

f) $f_6(x) = \frac{2x^3}{(x^5-x^4)^3}$·; $\mathbb{D} = x \in \mathbb{R}\backslash\{0;1\}$

$$f_6{}'(x) = \frac{6x^2\cdot(x^5-x^4)^3-6x^3\cdot(x^5-x^4)^2\cdot(5x^4-4x^3)}{(x^5-x^4)^6} = \frac{(x^5-x^4)^2\cdot\left(6x^2\cdot(x^5-x^4)-6x^3\cdot(5x^4-4x^3)\right)}{(x^5-x^4)^6}$$
$$= \frac{6x^7-6x^6-30x^7+24x^6}{(x^5-x^4)^4} = \frac{-24x^7+18x^6}{x^{16}(x-1)^4} = -\frac{24x-18}{x^{10}\cdot(x-1)^4}$$

g) $f_7(x) = \frac{3x-x^5}{(5x)^4}$; $\mathbb{D} = x \in \mathbb{R}\backslash\{0\}$

$$f_7{}'(x) = \frac{(3-5x^4)\cdot(5x)^4-20\cdot(5x)^3\cdot(3x-x^5)}{(5x)^8} = \frac{(5x)^3\cdot\left(5x\cdot(3-5x^4)-20\cdot(3x-x^5)\right)}{(5x)^3\cdot(5x)^5}$$
$$= \frac{15x-25x^5-60x+20x^5}{(5x)^5} = \frac{-5x^5-45x}{(5x)^5} = -\frac{5x(x^4+9)}{(5x)^5} = -\frac{x^4+9}{(5x)^4}$$

h) $f_8(x) = \frac{1}{(x-2)^2}$; $\mathbb{D} = x \in \mathbb{R}\backslash\{2\}$

$$f_8{}'(x) = \frac{-2\cdot(x-2)}{(x-2)^4} = -\frac{2}{(x-2)^3}$$

Lösung A3

$f(x) = \frac{4-x}{2-x}$ $\qquad f'(x) = \frac{-(2-x)+(4-x)}{(2-x)^2} = \frac{2}{(2-x)^2}$

$f(-2) = \frac{6}{4} = \frac{3}{2}$ $\qquad f'(-2) = \frac{1}{8}$

$f(0) = 2$ $\qquad f'(0) = \frac{1}{2}$

$f(1{,}5) = 5$ $\qquad f'(1{,}5) = 8$

$f(2{,}5) = -3$ $\qquad f'(2{,}5) = -8$

$f(6) = \frac{1}{2}$ $\qquad f'(6) = \frac{1}{8}$