Lösung A1

$f_1(x)=\frac{3x-2}{x^2-1}$	$f_1'(x)=\frac{3\cdot(x^2-1)-2x(3x-2)}{(x^2-1)^2}=\frac{-3x^2+4x-3}{(x^2-1)^2}=-\frac{3x^2-4x+3}{(x^2-1)^2}$
$f_2(x)=\frac{3-x^2}{x^2-2}$	$f_2'(x)=\frac{-2x\cdot(x^2-2)-2x\cdot(3-x^2)}{(x^2-2)^2}=-\frac{2x}{(x^2-2)^2}$
$f_3(x)=\frac{(x-1)^2}{(x+1)^2}$	$f_3'(x)=\frac{2(x-1)(x+1)^2-2(x+1)(x-1)^2}{(x+1)^4}=\frac{2(x+1)(x^2-1-x^2+2x-1)}{(x+1)^4}$ $=\frac{2(x+1)\cdot 2(x-1)}{(x+1)^4}=\frac{4(x-1)}{(x+1)^3}$
$f_4(x)=\frac{2x^3-2x}{x}$	$f_4'(x)=4x$
$f_5(x)=\frac{5x^2-3}{-x^2}$	$f_5'(x)=\frac{-10x^3+2x(5x^2-3)}{x^4}=\frac{-6}{x^3}$
$f_6(x)=\frac{x^2-3x}{4x^3}$	$f_6'(x)=\frac{4x^3\cdot(2x-3)-12x^2\cdot(x^2-3x)}{16x^6}=\frac{x(2x-3)-3(x^2-3x)}{4x^4}=\frac{-x^2+6x}{4x^4}=-\frac{x-6}{4x^3}$
$f_7(x)=\frac{(x-20)^5}{(x+10)^{-2}}$	$f_7'(x)=\frac{5(x-20)^4\cdot(x+10)^{-2}+2(x+10)^{-3}\cdot(x-20)^5}{(x+10)^{-4}}$ $=5(x-20)^4\cdot(x+10)^2+2(x-20)^5(x+10)$ $=(x-20)^4\cdot(x+10)\cdot(7x+10)$
$f_8(x)=\frac{x^2-x+2}{\sin(t)}\cdot$	$f_8'(x)=\frac{1}{\sin(t)}\cdot(2x-1)$
$f_9(x)=\frac{3x^3+4}{x^2}$	$f_9'(x)=3-\frac{8}{x^3}$

Lösung A2

$f_1'(x)=\frac{1}{2\cdot\sqrt{x-1}\cdot(x+1)^{\frac{3}{2}}}$	$f_{18}(x)=\frac{0{,}5\cdot(x-1)^{\frac{1}{2}}}{(x+1)^{\frac{1}{2}}}$
$f_2'(x)=1+tan^2(x)$	$f_{17}(x)=\frac{sin(x)}{cos(x)}$
$f_3'(x)=-\frac{\pi\cdot sin(x-3)}{x^2-2}-\frac{2\pi x\cdot cos(x-3)}{(x^2-2)^2}$	$f_{16}(x)=\frac{\pi cos(x-3)}{x^2-2}$
$f_4'(x)=\frac{6}{(3x-4)^3}$	$f_{15}(x)=\frac{(4-3x)^{-1}}{3x-4}$
$f_5'(x)=-\frac{3x^4\cdot(x+5)}{(x+3)^2}$	$f_{14}(x)=\frac{-x^5}{(x+3)^2}$
$f_6'(x)=\frac{10x-4}{a}$	$f_{13}(x)=\frac{5x^4-4x^3}{ax^2}$
$f_7'(x)=\frac{10x^3+x^2+4}{x^2}$	$f_{12}(x)=\frac{5x^3+x^2-4}{x}$
$f_8'(x)=\frac{(2x-2)\cdot\sin(x+2)+(2x-x^2)\cdot\cos(x+2)}{\sin^2(x+2)}$	$f_{11}(x)=\frac{x^2-2x}{sin(x+2)}$
$f_9'(x)=4$	$f_{10}(x)=\frac{4x^2-3x}{x}$

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Lösung A3

Aa) $f(x)=\frac{2x^3}{4x^2}$ $\quad u=2x^3 \quad u'=6x^2 \quad v=4x^2 \quad v'=8x$

$f'(x)=\frac{24x^4-16x^4}{16x^4}=\frac{1}{2}$

$f''(x)=0$

b) $f(x)=\frac{x^3}{2x^2-4}$ $\quad u=x^3 \quad u'=3x^2 \quad v=2x^2-4 \quad v'=4x$

$f'(x)=\frac{6x^2\cdot(x^2-2)-4x^4}{4(x^2-2)^2}=\frac{x^4-6x^2}{2(x^2-2)^2}$ $\quad u=x^4-6x^2 \quad u'=4x^3-12x$

$v=2(x^2-2)^2 \quad v'=8x(x^2-2)$

$f''(x)=\frac{4\cdot(x^3-3x)\cdot 2\cdot(x^2-2)^2-(x^4-6x^2)\cdot 8x\cdot(x^2-2)}{4(x^2-2)^4}=\frac{8\cdot(x^2-2)\cdot\left((x^3-3x)\cdot(x^2-2)-x(x^4-6x^2)\right)}{4(x^2-2)^4}$

$=\frac{2\cdot\left((x^3-3x)\cdot(x^2-2)-x(x^4-6x^2)\right)}{(x^2-2)^3}=\frac{2\cdot(x^5-2x^3-3x^3+6x-x^5+6x^3)}{(x^2-2)^3}$

$=\frac{2x\cdot(x^2+6)}{(x^2-2)^3}$

c) $f(x)=\frac{2(x^3-x)}{x^2+x}$ $\quad u=2(x^3-x) \quad u'=2(3x^2-1)$

$v=x^2+x \quad v'=2x+1$

$f'(x)=\frac{2(3x^2-1)(x^2+x)-2(x^3-x)(2x+1)}{(x^2+x)^2}=\frac{2\left(3x^4+3x^3-x^2-x-(2x^4+x^3-2x^2-x)\right)}{(x^2+x)^2}$

$=\frac{2(x^4+2x^3+x^2)}{(x^2+x)^2}=\frac{2(x^4+2x^3+x^2)}{x^4+2x^3+x^2}=2$

$f''(x)=0$

d) $f(x)=\frac{x-1}{(x-k)^2}$ $\quad u=x-1 \quad u'=1 \quad v=(x-k)^2 \quad v'=2(x-k)$

$f'(x)=\frac{(x-k)^2-2\cdot(x-1)\cdot(x-k)}{(x-k)^4}=\frac{-x-k+2}{(x-k)^3}=-\frac{x+k-2}{(x-k)^3}$

$u=-x-k+2 \quad u'=-1$

$v=(x+k)^3 \quad v'=3(x+k)^2$

$f''(x)=\frac{-(x-k)^3-(-x-k+2)\cdot 3\cdot(x-k)^2}{(x-k)^6}=\frac{-(x-k)+3\cdot(x+k-2)}{(x-k)^4}=\frac{2x+4k-6}{(x-k)^4}=\frac{2(x+2k-3)}{(x-k)^4}$

e) $f(x)=\frac{2ax}{(x-a)^2}$ $\quad u=2ax \quad u'=2a \quad v=(x-a)^2 \quad v'=2(x-a)$

$f'(x)=\frac{2a(x-a)^2-4ax(x-a)}{(x-a)^4}=\frac{2a(x-a)-4ax}{(x-a)^3}=\frac{-2ax-2a^2}{(x-a)^3}=-\frac{2a(x+a)}{(x-a)^3}$

$u=-2a(x+a) \quad u'=-2a$

$v=(x-a)^3 \quad v'=3(x-a)^2$

$f''(x)=\frac{-2a(x-a)^3+6a(x+a)(x-a)^2}{(x-a)^6}=\frac{-2a(x-a)+6a(x+a)}{(x-a)^4}=\frac{4ax+8a^2}{(x-a)^4}=\frac{4a(x+2a)}{(x-a)^4}$

f) $f(x)=\frac{2x}{x^2+2}$ $\quad u=2x \quad u'=2 \quad v=x^2+2 \quad v'=2x$

$f'(x)=\frac{2(x^2+2)-4x^2}{(x^2+2)^2}=\frac{2(2-x^2)}{(x^2+2)^2}$

$u=4-2x^2 \quad u'=-4x$

$v=(x^2+2)^2 \quad v'=4x\cdot(x^2+2)$

$f''(x)=\frac{-4x\cdot(x^2+2)^2-4x\cdot(x^2+2)\cdot(4-2x^2)}{(x^2+2)^4}=\frac{-4x(x^2+2)-4x(4-2x^2)}{(x^2+2)^3}=\frac{4x^3-24x}{(x^2+2)^2}=\frac{4x(x^2-6)}{(x^2+2)^2}$

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