

Aufgabenblatt Ableitungen

zur Produkt- und Quotientenregel

Lösungen

Level 3 – Expert – Blatt 1

Lösung A1

a) $f(x) = (x^2 - 4) \cdot (x^3 + 1)$ $u = x^2 - 4$ $u' = 2x$ $v = x^3 + 1$ $v' = 3x^2$
 $f'(x) = 2x \cdot (x^3 + 1) + 3x^2 \cdot (x^2 - 4) = 2x^4 + 2x + 3x^4 - 12x^2 = x \cdot (5x^3 - 12x + 2)$

b) $f(x) = \left(\frac{1}{2}x - 1\right) \cdot (4 - 0,8x^2)$ $u = \frac{1}{2}x - 1$ $u' = \frac{1}{2}$
 $v = 4 - \frac{4}{5}x^2$ $v' = -\frac{8}{5}x$
 $f'(x) = \frac{1}{2} \cdot \left(4 - \frac{4}{5}x^2\right) - \frac{8}{5}x \cdot \left(\frac{x}{2} - 1\right) = 2 - \frac{2}{5}x^2 - \frac{4}{5}x^2 + \frac{8}{5}x$
 $= \frac{6x^2 - 8x - 10}{5}$

c) $f(t) = (3t^2 + 1) \cdot (1 - t^2)$ $u = 3t^2 + 1$ $u' = 6t$
 $v = 1 - t^2$ $v' = -2t$
 $f'(t) = 6t \cdot (1 - t^2) - 2t \cdot (3t^2 + 1) = 2t \cdot (3 - 3t^2 - 3t^2 - 1)$
 $= 4t \cdot (1 - 3t^2)$

d) $f(x) = (x^3 + x^2) \cdot (1 - x)$ $u = x^3 + x^2$ $u' = 3x^2 + 2x$
 $v = 1 - x$ $v' = -1$
 $f'(x) = (3x^2 + 2x) \cdot (1 - x) - (x^3 + x^2) = 3x^2 - 3x^3 + 2x - 2x^2 - x^3 - x^2$
 $= -4x^3 + 2x = -2x \cdot (2x^2 - 1)$

e) $f(r) = (1 + r^2)^2$
 $f'(r) = 2(1 + r^2) \cdot 2r = 4r \cdot (r^2 + 1)$

f) $f(x) = x^2 \cdot \sqrt{x} = x^{\frac{5}{2}}$
 $f'(x) = \frac{5}{2}x^{\frac{3}{2}} = \frac{5}{2}x \cdot \sqrt{x}$

Lösung A2

a) $f(x) = \sqrt{x} \cdot (2x - 1)$ $u = \sqrt{x}$ $u' = \frac{1}{2\sqrt{x}}$
 $v = 2x - 1$ $v' = 2$
 $f'(x) = \frac{1}{2\sqrt{x}} \cdot (2x - 1) + 2\sqrt{x} = \frac{2x-1+4x}{2\sqrt{x}} = \frac{6x-1}{2\sqrt{x}}$
 $u = \frac{6x-1}{2}$ $u' = 3$
 $v = x^{-\frac{1}{2}}$ $v' = -\frac{1}{2}x^{-\frac{3}{2}}$
 $f''(x) = \frac{3}{\sqrt{x}} - \frac{6x-1}{4x\sqrt{x}} = \frac{12x-6x+1}{4x\sqrt{x}} = \frac{6x+1}{4\sqrt{x}}$

b) $f(t) = (4t^2 - 1) \cdot \sqrt{t}$ $u = 4t^2 - 1$ $u' = 8t$
 $v = \sqrt{t}$ $v' = \frac{1}{2\sqrt{t}}$
 $f'(t) = 8t \cdot \sqrt{t} + \frac{1}{2\sqrt{t}} \cdot (4t^2 - 1) = 8t^{\frac{3}{2}} + 2t^{\frac{3}{2}} - \frac{1}{2\sqrt{t}} = \frac{20t^2 - 1}{2\sqrt{t}}$
 $u = 20t^2 - 1$ $u' = 40t$
 $v = \frac{1}{2}t^{-\frac{1}{2}}$ $v' = -\frac{1}{4t\sqrt{t}}$
 $f''(t) = \frac{20t}{\sqrt{t}} - \frac{20t^2 - 1}{4t\sqrt{t}} = \frac{80t^2 - 20t^2 + 1}{4t\sqrt{t}} = \frac{60t^2 + 1}{4t\sqrt{t}}$

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Differenzialrechnung Lösungen

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- c) $f(a) = \sqrt{a} \cdot (1 - a)$ $u = \sqrt{a}$ $u' = \frac{1}{2\sqrt{a}}$
 $v = 1 - a$ $v' = -1$
- $$f'(a) = \frac{1}{2\sqrt{a}}(1 - a) - \sqrt{a} = \frac{1-a-2a}{2\sqrt{a}} = -\frac{3a-1}{2\sqrt{a}}$$
- $$u = -(3a - 1) \quad u' = -3$$
- $$v = \frac{1}{2}a^{-\frac{1}{2}} \quad v' = -\frac{1}{4a\sqrt{a}}$$
- $$f''(a) = -\frac{3}{2\sqrt{a}} + \frac{3a-1}{4a\sqrt{a}} = \frac{-6a+3a-1}{4a\sqrt{a}} = -\frac{3a+1}{4a\sqrt{a}}$$
- d) $f(z) = (z^2 - 1) \cdot \sqrt{z}$ $u = z^2 - 1$ $u' = 2z$
 $v = \sqrt{z}$ $v' = \frac{1}{2\sqrt{z}}$
- $$f'(z) = 2z\sqrt{z} + \frac{z^2-1}{2\sqrt{z}} = \frac{4z^2+z^2-1}{2\sqrt{z}} = \frac{5z^2-1}{2\sqrt{z}}$$
- $$u = 5z^2 - 1 \quad u' = 10z$$
- $$v = \frac{1}{2}z^{-\frac{1}{2}} \quad v' = -\frac{1}{4z\sqrt{z}}$$
- $$f''(z) = \frac{10z}{2\sqrt{z}} - \frac{5z^2-1}{4z\sqrt{z}} = \frac{20z^2-5z^2+1}{4z\sqrt{z}} = \frac{15z^2+1}{4z\sqrt{z}}$$
- e) $f(t) = \sin(t) \cdot \cos(t)$ $u = \sin(t)$ $u' = \cos(t)$
 $v = \cos(t)$ $v' = -\sin(t)$
- $$f'(t) = \cos^2(t) - \sin^2(t)$$
- $$f''(t) = -2 \cos(t) \sin(t) - 2 \sin(t) \cos(t) = -4 \sin(t) \cos(t)$$
- f) $f_a(t) = a(\sin(at) \cdot \cos(at) \cdot t^2) = at^2 \cdot \sin(at) \cdot \cos(at)$
Wegen des Additionstheorems $\sin(2\alpha) = 2\sin(\alpha) \cdot \cos(\alpha)$ ist
 $(\sin(at) \cdot \cos(at)) = \sin(2at)$ und damit
- $$f_a(t) = at^2 \cdot \sin(2at) \quad u = at^2 \quad u' = 2at$$
- $$v = \sin(2at) \quad v' = 2a\cos(2at)$$
- $$f_a'(t) = 2at \cdot \sin(2at) + 2a^2t^2 \cdot \cos(2at)$$
- $$= 2at \cdot (\sin(2at) + at\cos(2at))$$
- $$f_a''(t) = 2at(3a\cos(2at) - 2a^2t \cdot \sin(2at)) + 2a \cdot (\sin(2at) + at \cdot \cos(2at))$$
- $$= (2a - 4a^3t^2)\sin(2at) + 8a^2t\cos(2at)$$

Lösung A3

- a) $f_1(x) = \frac{1}{x} \cdot (1 - x^3)$ $u = \frac{1}{x}$ $u' = -\frac{1}{x^2}$
 $v = 1 - x^3$ $v' = -3x^2$
- $$f_1'(x) = -\frac{(1-x^3)}{x^2} - \frac{3x^2}{x} = \frac{x^3-1-3x^3}{x^2} = -\frac{2x^3+1}{x^2} \quad f_1'(-1) = 1$$
- b) $f_2(x) = x \cdot (t^2 - t)$ $f_2'(2) = t^2 - t$
- $$f_2'(x) = t^2 - t$$
- c) $f_3(x) = \sin(x) \cdot (x^2 + 1)$ $u = \sin(x)$ $u' = \cos(x)$
 $v = x^2 + 1$ $v' = 2x$
- $$f_3'(x) = \cos(x) \cdot (x^2 + 1) + 2x \cdot \sin(x) \quad f_3'\left(\frac{\pi}{2}\right) = \pi$$
- d) $f_4(x) = \sin(2x) \cdot \cos(x)$ $u = \sin(2x)$ $u' = 2\cos(2x)$
 $v = \cos(x)$ $v' = -\sin(x)$
- $$f_4'(x) = 2\cos(2x) \cdot \cos(x) - 2\sin(2x) \cdot \sin(x)$$
- $$f_4'(0) = 2$$

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Lösung A4

a) $f(x) = \sqrt{2 + (x - 1)^2}$

$$f'(x) = \frac{-2(x-1)}{2\sqrt{2+(x-1)^2}} = \frac{x-1}{\sqrt{2+(x-1)^2}}$$

$$g(x) = x^2 - 2x + 3$$

$$g'(x) = 2x - 2$$

$$f'(x) \cap g'(x)$$

$$\frac{x-1}{\sqrt{2+(x-1)^2}} = 2(x-1)$$

$$x_0 = 1$$

$$g'(1) = 0$$

An der Stelle $x_0 = 1$ verlaufen die beiden Graphen mit einer Steigung von $m = 0$ parallel.

b) $f(x) = (\sin(2x))^2$

$$f'(x) = 4\sin(2x) \cdot \cos(2x)$$

$$g(x) = \sin(1 - \sqrt{x})$$

$$g'(x) = -\frac{\cos(\sqrt{x}-1)}{2\sqrt{x}}$$

$$f'(x) \cap g'(x)$$

$$4\sin(2x) \cdot \cos(2x) = -\frac{\cos(\sqrt{x}-1)}{2\sqrt{x}}$$

$$4\sin(2x) \cdot \cos(2x) + \frac{\cos(\sqrt{x}-1)}{2\sqrt{x}} = 0$$

$$x_0 \approx 0,85; f'(0,85) = g'(0,85) \approx -0,54$$

An der Stelle $x_0 \approx 0,85$ verlaufen die beiden Graphen mit einer Steigung von $m \approx -0,54$ parallel.