



## Aufgabe A1

Bilde die 1. und 2. Ableitung der gegebenen Funktionsgleichungen mit Hilfe der Quotientenregel. Beachte, dass du in manchen Fällen auch die Kettenregel benötigst. Vereinfache die 1. Ableitung bevor du die 2. Ableitung bildest.

- |   |                                     |
|---|-------------------------------------|
| a) $f(x) = \frac{\sqrt{x}}{(x-1)^4}$        | b) $f(x) = \frac{x^2}{(1-x)^5}$     |
| c) $f(x) = \frac{\sqrt{x}}{5+\sqrt{x}}$     | d) $f(x) = \frac{3x^5-2x}{\sin(x)}$ |
| e) $f(x) = \frac{\sqrt{x}}{5x+\frac{1}{x}}$ | f) $f(x) = \frac{5\sqrt{x}+3x}{5x}$ |

## Aufgabe A2

Bilde die 1. Ableitung der gegebenen Funktionsgleichungen mit Hilfe der Quotientenregel. Beachte, dass du in manchen Fällen auch die Kettenregel benötigst. Vereinfache dein Ergebnis so weit wie möglich.

- |   |   |
|---|---|
| a) $f(x) = \frac{\frac{2}{3}x^3}{5x^2-x+7}$ | b) $f(x) = \frac{\frac{2}{x^2}-1}{x+5}$                   |
| c) $f(x) = \frac{5\sqrt{x}+3x}{5x+7}$       | d) $f(x) = \frac{-\frac{1}{x^2}+\frac{1}{10}x}{\sqrt{x}}$ |
| e) $f(x) = \frac{7x^5+x^2-2x}{x^7+3x}$      | f) $f_a(t) = \frac{\sin(at)+at}{t^2}$                     |

## Aufgabe A3

Berechne die Steigung der Funktionen  $f_n$  an der angegebenen Stelle  $x_0$ .

- |  |   |
|--|---|
| a) $f_1(x) = \frac{\frac{1}{x}+1}{3x+5}; x_0 = -1$         | b) $f_2(x) = \frac{2x^6}{\frac{1}{x^3}-\frac{1}{x^4}}; x_0 = 2$           |
| c) $f_3(x) = \frac{\sqrt{x}}{5x^2+\frac{1}{x^2}}; x_0 = 1$ | d) $f_4(x) = \frac{\frac{1}{8}x^3+\frac{6}{5}x}{\frac{2}{5}x-8}; x_0 = 0$ |

## Aufgabe A4

An welcher Stelle verlaufen die Graphen der Funktionen  $f$  und  $g$  parallel? Welche Steigung haben die Tangenten an dieser Stelle?

- |  |                           |
|--|---------------------------|
| a) $f(x) = \frac{x+3x^2}{2x-1}$              | $g(x) = \frac{1}{(2x-1)}$ |
| b) $f(x) = \frac{\frac{1}{2}x^2+8x}{x^2-4x}$ | $g(x) = 3x+2$             |

### Lösung A1

a)  $f(x) = \frac{\sqrt{x}}{(x-1)^4}$        $u = \sqrt{x}$        $u' = \frac{1}{2\sqrt{x}}$        $v = (x-1)^4$        $v' = 4(x-1)^3$

$$f'(x) = \frac{\frac{1}{2\sqrt{x}}(x-1)^4 - 4\sqrt{x}(x-1)^3}{(x-1)^8} = \frac{(x-1)^3 \cdot \left(\frac{x-1}{2\sqrt{x}} - 4\sqrt{x}\right)}{(x-1)^8} = \frac{\left(\frac{x-1}{2\sqrt{x}} - 4\sqrt{x}\right)}{(x-1)^5}$$

$$= -\frac{2 \cdot (7x+1) \cdot x^{\frac{1}{2}}}{(x-1)^5} = -\frac{7x+1}{2\sqrt{x}(x-1)^5} \quad u = -(7x+1) \quad u' = -7$$

$$v = 2\sqrt{x} \cdot (x-1)^5 \quad v' = \frac{(x-1)^5}{\sqrt{x}} + 10\sqrt{x} \cdot (x-1)^4$$

$$f''(x) = \frac{-14\sqrt{x}(x-1)^5 + (7x+1) \cdot \left((x-1)^4 \left(10\sqrt{x} + \frac{x-1}{\sqrt{x}}\right)\right)}{4x \cdot (x-1)^{10}} = \frac{(x-1)^4 \cdot \left(-14\sqrt{x}(x-1) + (7x+1) \left(10\sqrt{x} + \frac{x-1}{\sqrt{x}}\right)\right)}{4x \cdot (x-1)^{10}}$$

$$= \frac{-14x\sqrt{x} + 14\sqrt{x} + 70x\sqrt{x} + 7\sqrt{x}(x-1) + 10\sqrt{x} + \frac{x-1}{\sqrt{x}}}{4x \cdot (x-1)^6} = \frac{56x\sqrt{x} + 14\sqrt{x} + 7x\sqrt{x} - 7\sqrt{x} + 10\sqrt{x} + \frac{x-1}{\sqrt{x}}}{4x \cdot (x-1)^6}$$

$$= \frac{63x\sqrt{x} + 17\sqrt{x} + \frac{x-1}{\sqrt{x}}}{4x \cdot (x-1)^6} = \frac{63x^2 + 17x + x - 1}{4x \cdot \sqrt{x} \cdot (x-1)^6} = \frac{63x^2 + 18x - 1}{4x \cdot \sqrt{x} \cdot (x-1)^6}$$

b)  $f(x) = \frac{x^2}{(1-x)^5}$        $u = x^2$        $u' = 2x$

$$v = (1-x)^5 \quad v' = -5(1-x)^4$$

$$f'(x) = \frac{2x \cdot (1-x)^5 + 5x^2 \cdot (1-x)^4}{(1-x)^{10}} = \frac{(1-x)^4 \cdot (5x^2 + 2x - 2x^2)}{(1-x)^{10}}$$

$$= \frac{3x^2 + 2x}{(x-1)^6} \quad u = 3x^2 + 2x \quad u' = 6x + 2$$

$$v = (x-1)^6 \quad v' = 6(x-1)^5$$

$$f''(x) = \frac{(6x+2) \cdot (x-1)^6 + 6 \cdot (x-1)^5 \cdot (3x^2+2x)}{(1-x)^{12}} = \frac{(x-1)^5 \cdot ((6x+2) \cdot (x-1) + 6 \cdot (3x^2+2x))}{(1-x)^{12}}$$

$$= \frac{6x - 6x^2 + 2 - 2x + 18x^2 + 12x}{(x-1)^7} = \frac{12x^2 + 16x + 2}{(x-1)^7}$$

c)  $f(x) = \frac{\sqrt{x}}{5+\sqrt{x}}$        $u = \sqrt{x}$        $u' = \frac{1}{2\sqrt{x}}$

$$v = 5 + \sqrt{x} \quad v' = \frac{1}{2\sqrt{x}}$$

$$f'(x) = \frac{\frac{1}{2\sqrt{x}}(5+\sqrt{x}) - \frac{1}{2\sqrt{x}} \cdot \sqrt{x}}{(5+\sqrt{x})^2} = \frac{\frac{5}{2\sqrt{x}}}{(5+\sqrt{x})^2} = \frac{5}{2\sqrt{x}(5+\sqrt{x})^2}$$

$$u = 5 \quad u' = 0$$

$$v = 2\sqrt{x}(5+\sqrt{x})^2 \quad v' = \frac{1}{\sqrt{x}}(5+\sqrt{x})^2 + 2(5+\sqrt{x})$$

$$f''(x) = -\frac{\frac{5}{\sqrt{x}}(5+\sqrt{x})^2 + 10(5+\sqrt{x})}{4x(5+\sqrt{x})^4} = -\frac{(5+\sqrt{x}) \cdot \left(\frac{5}{\sqrt{x}}(5+\sqrt{x}) + 10\right)}{4x(5+\sqrt{x})^4} = -\frac{\frac{25}{\sqrt{x}} + 15}{4x(5+\sqrt{x})^3}$$

$$= -\frac{25 + 15\sqrt{x}}{4x\sqrt{x}(5+\sqrt{x})^3}$$

d)  $f(x) = \frac{3x^5 - 2x}{\sin(x)}$        $u = 3x^5 - 2x$        $u' = 15x^4 - 2$

$$v = \sin(x) \quad v' = \cos(x)$$

$$f'(x) = \frac{(15x^4 - 2) \cdot \sin(x) - (3x^5 - 2x) \cdot \cos(x)}{\sin^2(x)}$$

$$u = (15x^4 - 2) \cdot \sin(x) - (3x^5 - 2x) \cdot \cos(x)$$

$$u' = 60x^3 \cdot \sin(x) + (15x^4 - 2) \cdot \cos(x) - (15x^4 - 2) \cdot \cos(x) + (3x^5 - 2x) \cdot \sin(x)$$

$$= \sin(x) \cdot (60x^3 + 3x^5 - 2x) \quad v = \sin^2(x) \quad v' = 2\sin(x)\cos(x)$$

$$\begin{aligned}
 f''(x) &= \frac{\sin^2(x) \cdot (3x^5 + 60x^3 - 2x) - (30x^4 - 4) \cdot \sin^2(x) \cdot \cos(x) + (6x^5 - 4x) \sin(x) \cos^2(x)}{\sin^4(x)} \\
 &= \frac{\sin(x) \cdot (\sin(x) \cdot (3x^5 + 60x^3 - 2x) - (30x^4 - 4) \cdot \sin(x) \cdot \cos(x) + (6x^5 - 4x) \cdot \cos^2(x))}{\sin^4(x)} \\
 &= \frac{\sin(x) \cdot (3x^5 + 60x^3 - 2x) - (30x^4 - 4) \cdot \sin(x) \cdot \cos(x) + (6x^5 - 4x) \cdot \cos^2(x)}{\sin^3(x)}
 \end{aligned}$$

e)  $f(x) = \frac{\sqrt{x}}{5x + \frac{1}{x}}$        $u = \sqrt{x}$        $u' = \frac{1}{2\sqrt{x}}$   
 $v = 5x + \frac{1}{x}$        $v' = 5 - \frac{1}{x^2}$

$$\begin{aligned}
 f'(x) &= \frac{\frac{1}{2\sqrt{x}} \cdot (5x + \frac{1}{x}) - \sqrt{x} \cdot (5 - \frac{1}{x^2})}{(5x + \frac{1}{x})^2} = \frac{\frac{1}{\sqrt{x}}(5x + \frac{1}{x}) - 2\sqrt{x}(5 - \frac{1}{x^2})}{2 \cdot (5x + \frac{1}{x})^2} = \frac{\frac{1}{\sqrt{x}}(5x + \frac{1}{x}) - 2\sqrt{x}(5 - \frac{1}{x^2})}{2 \cdot (\frac{5x^2 + 1}{x})^2} \\
 &= \frac{x^2 \cdot (\frac{1}{\sqrt{x}}(5x + \frac{1}{x}) - 2\sqrt{x}(5 - \frac{1}{x^2}))}{2 \cdot (5x^2 + 1)^2} = \frac{5x^2\sqrt{x} + \sqrt{x} - 10x^2\sqrt{x} + 2\sqrt{x}}{2 \cdot (5x^2 + 1)^2} = \frac{-5x^2\sqrt{x} + 3\sqrt{x}}{2 \cdot (5x^2 + 1)^2} \\
 &= -\frac{\sqrt{x}(5x^2 - 3)}{2 \cdot (5x^2 + 1)^2} \quad u = \sqrt{x}(3 - 5x^2) \quad u' = \frac{3}{2\sqrt{x}} - \frac{25}{2}x\sqrt{x} \\
 &\quad v = 2 \cdot (5x^2 + 1)^2 \quad v' = 40x \cdot (5x^2 + 1)
 \end{aligned}$$

$$\begin{aligned}
 f''(x) &= \frac{2 \cdot \left( \frac{3}{2\sqrt{x}} - \frac{25}{2}x\sqrt{x} \right) \cdot (5x^2 + 1)^2 - 40x\sqrt{x} \cdot (5x^2 + 1) \cdot (3 - 5x^2)}{4 \cdot (5x^2 + 1)^4} \\
 &= \frac{(5x^2 + 1) \cdot \left( \left( \frac{3}{\sqrt{x}} - 25x\sqrt{x} \right) \cdot (5x^2 + 1) - 40x\sqrt{x} \cdot (3 - 5x^2) \right)}{4 \cdot (5x^2 + 1)^4} \\
 &= \frac{\left( \frac{3}{\sqrt{x}} - 25x\sqrt{x} \right) \cdot (5x^2 + 1) - 40x\sqrt{x} \cdot (3 - 5x^2)}{4 \cdot (5x^2 + 1)^3} = \frac{\frac{15x^2}{\sqrt{x}} - 125x^3\sqrt{x} + \frac{3}{\sqrt{x}} - 25x\sqrt{x} - 120x\sqrt{x} + 200x^3\sqrt{x}}{4 \cdot (5x^2 + 1)^3} \\
 &= \frac{15x^2 - 125x^4 + 3 - 25x^2 - 120x^2 + 200x^4}{4\sqrt{x} \cdot (5x^2 + 1)^3} = \frac{75x^4 - 130x^2 + 3}{4\sqrt{x} \cdot (5x^2 + 1)^3}
 \end{aligned}$$

f)  $f(x) = \frac{5\sqrt{x} + 3x}{5x} = \frac{1}{\sqrt{x}} + \frac{3}{5}$   
 $f'(x) = -\frac{1}{2x\sqrt{x}}$   
 $f''(x) = \frac{3}{4x^2\sqrt{x}}$

## Lösung A2

a)  $f(x) = \frac{\frac{2}{3}x^3}{5x^2 - x + 7}$        $u = \frac{2}{3}x^3$        $u' = 2x^2$   
 $v = 5x^2 - x + 7$        $v' = 10x - 1$

$$\begin{aligned}
 f'(x) &= \frac{2x^2 \cdot (5x^2 - x + 7) - \frac{2}{3}x^3 \cdot (10x - 1)}{(5x^2 - x + 7)^2} = \frac{10x^4 - 2x^3 + 14x^2 - \frac{20}{3}x^4 + \frac{2}{3}x^3}{(5x^2 - x + 7)^2} \\
 &= \frac{\frac{10}{3}x^4 - \frac{4}{3}x^3 + 14x^2}{(5x^2 - x + 7)^2} = \frac{10x^4 - 4x^3 + 42x^2}{3 \cdot (5x^2 - x + 7)^2}
 \end{aligned}$$

b)  $f(x) = \frac{\frac{2}{x^2} - 1}{x + 5}$        $u = \frac{2}{x^2} - 1$        $u' = -\frac{4}{x^3}$   
 $v = x + 5$        $v' = 1$

$$\begin{aligned}
 f'(x) &= \frac{-\frac{4}{x^3} \cdot (x + 5) - \frac{2}{x^2} + 1}{(x + 5)^2} = \frac{-\frac{4}{x^2} - \frac{20}{x^3} - \frac{2}{x^2} + 1}{(x + 5)^2} = \frac{-6x - 20 + x^3}{x^3 \cdot (x + 5)^2} = \frac{x^3 - 6x - 20}{x^3 \cdot (x + 5)^2} \\
 f''(x) &= \frac{60}{x^4} + \frac{4}{x^3}
 \end{aligned}$$

- c)  $f(x) = \frac{5\sqrt{x}+3x}{5x+7}$   $u = 5\sqrt{x} + 3x$   $u' = \frac{5}{2\sqrt{x}} + 3$   
 $v = 5x + 7$   $v' = 5$   
 $f'(x) = \frac{(\frac{5}{2\sqrt{x}}+3) \cdot (5x+7) - 5 \cdot (5\sqrt{x}+3x)}{(5x+7)^2} = \frac{\frac{25x}{2\sqrt{x}} + 15x + \frac{35}{2\sqrt{x}} + 21 - 25\sqrt{x} - 15x}{(5x+7)^2}$   
 $= \frac{\frac{25x+35+42\sqrt{x}-50x}{2\sqrt{x}}}{(5x+7)^2} = \frac{42\sqrt{x}-25x+35}{2\sqrt{x} \cdot (5x+7)^2}$
- d)  $f(x) = \frac{-\frac{1}{x^2} + \frac{1}{10}x}{\sqrt{x}}$   $u = -\frac{1}{x^2} + \frac{1}{10}x$   $u' = \frac{2}{x^3} + \frac{1}{10}$   
 $v = \sqrt{x}$   $v' = \frac{1}{2\sqrt{x}}$   
 $f'(x) = \frac{(\frac{2}{x^3} + \frac{1}{10}) \cdot \sqrt{x} - (-\frac{1}{x^2} + \frac{1}{10}x) \cdot \frac{1}{2\sqrt{x}}}{x} = \frac{2x \cdot (\frac{2}{x^3} + \frac{1}{10}) + \frac{1}{x^2} - \frac{1}{10}x}{2x\sqrt{x}}$   
 $= \frac{\frac{4}{x^2} + \frac{1}{10}x + \frac{1}{x^2}}{2x\sqrt{x}} = \frac{40+x^3+10}{20x^3\sqrt{x}} = \frac{x^3+50}{20x^3\sqrt{x}}$
- e)  $f(x) = \frac{7x^5+x^2-2x}{x^7+3x}$   $u = 7x^5 + x^2 - 2x$   $u' = 35x^4 + 2x - 2$   
 $v = x^7 + 3x$   $v' = 7x^6 + 3$   
 $f'(x) = \frac{(35x^4+2x-2) \cdot (x^7+3x) - (7x^5+x^2-2x) \cdot (7x^6+3)}{(x^7+3x)^2}$   
 $= \frac{35x^{11}+105x^5+2x^8+6x^2-2x^7-6x-(49x^{11}+7x^8-14x^7+21x^5+3x^2-6x)}{(x^7+3x)^2}$   
 $f'(x) = -\frac{14x^{11}+5x^8-12x^7-84x^5-3x^2}{(x^7+3x)^2} = -\frac{14x^9+5x^6-12x^5-84x^3-3}{(x^6+3)^2}$
- f)  $f_a(t) = \frac{\sin(at)+at}{t^2}$   $u = \sin(at) + at$   $u' = a \cdot \cos(at) + a$   
 $v = t^2$   $v' = 2t$   
 $f'_a(t) = \frac{(a \cdot \cos(at) + a) \cdot t^2 - 2t \cdot (\sin(at) + at)}{t^4} = \frac{t \cdot (at \cdot \cos(at) + at - 2 \sin(at) - 2at)}{t^4}$   
 $= \frac{at \cdot \cos(at) - 2 \sin(at) - at}{t^3}$

### Lösung A3

- a)  $f_1(x) = \frac{\frac{1}{x}+1}{3x+5}$   $u = \frac{1}{x} + 1$   $u' = -\frac{1}{x^2}$   
 $v = 3x + 5$   $v' = 3$   
 $f_1'(x) = \frac{-\frac{1}{x^2} \cdot (3x+5) - 3 \cdot (\frac{1}{x}+1)}{(3x+5)^2}$   $f_1'(-1) = -\frac{1}{2}$
- b)  $f_2(x) = \frac{2x^6}{\frac{1}{x^3} - \frac{1}{x^4}}$   $u = 2x^6$   $u' = 12x^5$   
 $v = \frac{1}{x^3} - \frac{1}{x^4}$   $v' = -\frac{3}{x^4} + \frac{4}{x^5}$   
 $f_2'(x) = \frac{12x^5 \cdot (\frac{1}{x^3} - \frac{1}{x^4}) - 2x^6 \cdot (-\frac{3}{x^4} + \frac{4}{x^5})}{(\frac{1}{x^3} - \frac{1}{x^4})^2}$   $f_2'(2) = 8192$
- c)  $f_3(x) = \frac{\sqrt{x}}{5x^2 + \frac{1}{x^2}}$   $u = \sqrt{x}$   $u' = \frac{1}{2\sqrt{x}}$   
 $v = 5x^2 + \frac{1}{x^2}$   $v' = 10x - \frac{2}{x^3}$   
 $f_3'(x) = \frac{\frac{1}{2\sqrt{x}} \cdot (5x^2 + \frac{1}{x^2}) - \sqrt{x} \cdot (10x - \frac{2}{x^3})}{(5x^2 + \frac{1}{x^2})^2}$   $f_3'(1) = -\frac{5}{36}$

d)  $f_4(x) = \frac{\frac{1}{8}x^3 + \frac{6}{5}x}{\frac{2}{5}x - 8}$   $u = \frac{1}{8}x^3 + \frac{6}{5}x$   $u' = \frac{3}{8}x^2 + \frac{6}{5}$   
 $v = \frac{2}{5}x - 8$   $v' = \frac{2}{5}$   
 $f_4'(x) = \frac{(\frac{3}{8}x^2 + \frac{6}{5}) \cdot (\frac{2}{5}x - 8) - \frac{2}{5} \cdot (\frac{1}{8}x^3 + \frac{6}{5}x)}{(\frac{2}{5}x - 8)^2}$   $f_4'(0) = -\frac{3}{20}$

### Lösung A4

a)  $f(x) = \frac{x+3x^2}{2x-1}$   $u = x + 3x^2$   $u' = 1 + 6x$   
 $v = 2x - 1$   $v' = 2$

$$f'(x) = \frac{(1+6x) \cdot (2x-1) - 2 \cdot (x+3x^2)}{(2x-1)^2} = \frac{6x^2 - 6x - 1}{(2x-1)^2}$$

$$g(x) = \frac{1}{(2x-1)}$$

$$g'(x) = -\frac{2}{(2x-1)^2}$$

$$f'(x) \cap g'(x)$$

$$\frac{6x^2 - 6x - 1}{(2x-1)^2} = -\frac{2}{(2x-1)^2}$$

$$6x^2 - 6x + 1 = 0$$

$$x^2 - x + \frac{1}{6} = 0$$

$$x_{1,2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{1}{6}} = -\frac{1}{2} \pm \sqrt{\frac{1}{12}} = -\frac{1}{2} \pm \frac{1}{2} \cdot \sqrt{\frac{1}{3}} = -\frac{1}{2} \pm \frac{1}{6}\sqrt{3}$$

$$m_1 = g'(-\frac{1}{2} + \frac{1}{6}\sqrt{3}) \approx -1 \quad m_2 = g'(-\frac{1}{2} - \frac{1}{6}\sqrt{3}) \approx -0,3$$

An den Stellen  $x_1 = -\frac{1}{2} + \frac{1}{6}\sqrt{3}$  und  $x_2 = -\frac{1}{2} - \frac{1}{6}\sqrt{3}$  verlaufen die beiden Graphen mit einer Steigung von  $m_1 \approx -1$  und  $m_2 \approx -0,3$  parallel.

b)  $f(x) = \frac{\frac{1}{2}x^2 + 8x}{x^2 - 4x}$   $u = \frac{1}{2}x^2 + 8x$   $u' = x + 8$   
 $v = x^2 - 4x$   $v' = 2x - 4$

$$f'(x) = \frac{(x+8) \cdot (x^2-4x) - (\frac{1}{2}x^2+8x) \cdot (2x-4)}{(x^2-4x)^2} = \frac{10}{(x-4)^2}$$

$$g(x) = 3x + 2$$

$$g'(x) = 3$$

$$f'(x) \cap g'(x)$$

$$\frac{10}{(x-4)^2} = 3$$

$$(x-4)^2 = \frac{10}{3}$$

$$|x-4| = \sqrt{\frac{10}{3}}$$

$$x_1 = 4 + \sqrt{\frac{10}{3}}; \quad x_2 = 4 - \sqrt{\frac{10}{3}}$$

An den Stellen  $x_1 = 4 + \sqrt{\frac{10}{3}}$  und  $x_2 = 4 - \sqrt{\frac{10}{3}}$  verlaufen die beiden Graphen mit einer Steigung von  $m = 3$  parallel.