

# Aufgabenblatt Ableitungen

## zur Produkt- und Quotientenregel

## Lösungen

Level 3 – Expert – Blatt 3

### Lösung A1

$$a) \quad f(x) = \frac{\sqrt{x}}{(x-1)^4} \quad u = \sqrt{x} \quad u' = \frac{1}{2\sqrt{x}} \quad v = (x-1)^4 \quad v' = 4(x-1)^3$$

$$f'(x) = \frac{\frac{1}{2\sqrt{x}}(x-1)^4 - 4\sqrt{x}(x-1)^3}{(x-1)^8} = \frac{(x-1)^3 \cdot \left(\frac{x-1}{2\sqrt{x}} - 4\sqrt{x}\right)}{(x-1)^8} = \frac{\left(\frac{x-1}{2\sqrt{x}} - 4\sqrt{x}\right)}{(x-1)^5}$$

$$= -\frac{2 \cdot (7x+1) \cdot x^{\frac{1}{2}}}{(x-1)^5} = -\frac{7x+1}{2\sqrt{x} \cdot (x-1)^5} \quad u = -(7x+1) \quad u' = -7$$

$$v = 2\sqrt{x} \cdot (x-1)^5 \quad v' = \frac{(x-1)^5}{\sqrt{x}} + 10\sqrt{x} \cdot (x-1)^4$$

$$f''(x) = \frac{-14\sqrt{x}(x-1)^5 + (7x+1) \cdot \left((x-1)^4 \cdot \left(10\sqrt{x} + \frac{x-1}{\sqrt{x}}\right)\right)}{4x \cdot (x-1)^{10}} = \frac{(x-1)^4 \cdot \left(-14\sqrt{x}(x-1) + (7x+1) \cdot \left(10\sqrt{x} + \frac{x-1}{\sqrt{x}}\right)\right)}{4x \cdot (x-1)^{10}}$$

$$= \frac{-14x\sqrt{x} + 14\sqrt{x} + 70x\sqrt{x} + 7\sqrt{x}(x-1) + 10\sqrt{x} + \frac{x-1}{\sqrt{x}}}{4x \cdot (x-1)^6} = \frac{56x\sqrt{x} + 14\sqrt{x} + 7x\sqrt{x} - 7\sqrt{x} + 10\sqrt{x} + \frac{x-1}{\sqrt{x}}}{4x \cdot (x-1)^6}$$

$$= \frac{63x\sqrt{x} + 17\sqrt{x} + \frac{x-1}{\sqrt{x}}}{4x \cdot (x-1)^6} = \frac{63x^2 + 17x + x-1}{4x \cdot \sqrt{x} \cdot (x-1)^6} = \frac{63x^2 + 18x - 1}{4x \cdot \sqrt{x} \cdot (x-1)^6}$$

$$b) \quad f(x) = \frac{x^2}{(1-x)^5} \quad u = x^2 \quad u' = 2x$$

$$v = (1-x)^5 \quad v' = -5(1-x)^4$$

$$f'(x) = \frac{2x \cdot (1-x)^5 + 5x^2 \cdot (1-x)^4}{(1-x)^{10}} = \frac{(1-x)^4 \cdot (5x^2 + 2x - 2x^2)}{(1-x)^{10}}$$

$$= \frac{3x^2 + 2x}{(x-1)^6} \quad u = 3x^2 + 2x \quad u' = 6x + 2$$

$$v = (x-1)^6 \quad v' = 6(x-1)^5$$

$$f''(x) = \frac{(6x+2) \cdot (x-1)^6 + 6 \cdot (x-1)^5 \cdot (3x^2 + 2x)}{(1-x)^{12}} = \frac{(x-1)^5 \cdot ((6x+2) \cdot (x-1) + 6 \cdot (3x^2 + 2x))}{(1-x)^{12}}$$

$$= \frac{6x - 6x^2 + 2 - 2x + 18x^2 + 12x}{(x-1)^7} = \frac{12x^2 + 16x + 2}{(x-1)^7}$$

$$c) \quad f(x) = \frac{\sqrt{x}}{5+\sqrt{x}} \quad u = \sqrt{x} \quad u' = \frac{1}{2\sqrt{x}}$$

$$v = 5 + \sqrt{x} \quad v' = \frac{1}{2\sqrt{x}}$$

$$f'(x) = \frac{\frac{1}{2\sqrt{x}} \cdot (5+\sqrt{x}) - \frac{1}{2\sqrt{x}} \cdot \sqrt{x}}{(5+\sqrt{x})^2} = \frac{\frac{5}{2\sqrt{x}}}{(5+\sqrt{x})^2} = \frac{5}{2\sqrt{x}(5+\sqrt{x})^2}$$

$$u = 5 \quad u' = 0$$

$$v = 2\sqrt{x}(5 + \sqrt{x})^2 \quad v' = \frac{1}{\sqrt{x}}(5 + \sqrt{x})^2 + 2(5 + \sqrt{x})$$

$$f''(x) = -\frac{\frac{5}{\sqrt{x}}(5+\sqrt{x})^2 + 10(5+\sqrt{x})}{4x(5+\sqrt{x})^4} = -\frac{(5+\sqrt{x}) \cdot \left(\frac{5}{\sqrt{x}}(5+\sqrt{x}) + 10\right)}{4x(5+\sqrt{x})^4} = -\frac{\frac{25}{\sqrt{x}} + 15}{4x(5+\sqrt{x})^3}$$

$$= -\frac{25 + 15\sqrt{x}}{4x\sqrt{x}(5+\sqrt{x})^3}$$

$$d) \quad f(x) = \frac{3x^5 - 2x}{\sin(x)} \quad u = 3x^5 - 2x \quad u' = 15x^4 - 2$$

$$v = \sin(x) \quad v' = \cos(x)$$

$$f'(x) = \frac{(15x^4 - 2) \cdot \sin(x) - (3x^5 - 2x) \cdot \cos(x)}{\sin^2(x)}$$

$$u = (15x^4 - 2) \cdot \sin(x) - (3x^5 - 2x) \cdot \cos(x)$$

$$u' = 60x^3 \cdot \sin(x) + (15x^4 - 2) \cdot \cos(x) - (15x^4 - 2) \cdot \cos(x) + (3x^5 - 2x) \cdot \sin(x)$$

$$= \sin(x) \cdot (60x^3 + 3x^5 - 2x) \quad v = \sin^2(x) \quad v' = 2\sin(x)\cos(x)$$

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$$\begin{aligned}
 f''(x) &= \frac{\sin^2(x) \cdot (3x^5 + 60x^3 - 2x) - (30x^4 - 4) \cdot \sin^2(x) \cdot \cos(x) + (6x^5 - 4x) \sin(x) \cos^2(x)}{\sin^4(x)} \\
 &= \frac{\sin(x) \cdot (\sin(x) \cdot (3x^5 + 60x^3 - 2x) - (30x^4 - 4) \cdot \sin(x) \cdot \cos(x) + (6x^5 - 4x) \cdot \cos^2(x))}{\sin^4(x)} \\
 &= \frac{\sin(x) \cdot (3x^5 + 60x^3 - 2x) - (30x^4 - 4) \cdot \sin(x) \cdot \cos(x) + (6x^5 - 4x) \cdot \cos^2(x)}{\sin^3(x)}
 \end{aligned}$$

e)  $f(x) = \frac{\sqrt{x}}{5x + \frac{1}{x}}$

$$\begin{aligned}
 u &= \sqrt{x} & u' &= \frac{1}{2\sqrt{x}} \\
 v &= 5x + \frac{1}{x} & v' &= 5 - \frac{1}{x^2} \\
 f'(x) &= \frac{\frac{1}{2\sqrt{x}} \cdot (5x + \frac{1}{x}) - \sqrt{x} \cdot (5 - \frac{1}{x^2})}{(5x + \frac{1}{x})^2} & & \\
 &= \frac{\frac{1}{\sqrt{x}}(5x + \frac{1}{x}) - 2\sqrt{x}(5 - \frac{1}{x^2})}{2 \cdot (5x + \frac{1}{x})^2} & & \\
 &= \frac{x^2 \cdot \left(\frac{1}{\sqrt{x}}(5x + \frac{1}{x}) - 2\sqrt{x}(5 - \frac{1}{x^2})\right)}{2 \cdot (5x^2 + 1)^2} & & \\
 &= \frac{5x^2\sqrt{x} + \sqrt{x} - 10x^2\sqrt{x} + 2\sqrt{x}}{2 \cdot (5x^2 + 1)^2} & & \\
 &= -\frac{\sqrt{x}(5x^2 - 3)}{2 \cdot (5x^2 + 1)^2} & u &= \sqrt{x}(3 - 5x^2) & u' &= \frac{3}{2\sqrt{x}} - \frac{25}{2}x\sqrt{x} \\
 & & v &= 2 \cdot (5x^2 + 1)^2 & v' &= 40x \cdot (5x^2 + 1) \\
 f''(x) &= \frac{2 \cdot \left(\left(\frac{3}{2\sqrt{x}} - \frac{25}{2}x\sqrt{x}\right) \cdot (5x^2 + 1)^2\right) - 40x\sqrt{x} \cdot (5x^2 + 1) \cdot (3 - 5x^2)}{4 \cdot (5x^2 + 1)^4} & & \\
 &= \frac{(5x^2 + 1) \cdot \left(\left(\frac{3}{\sqrt{x}} - 25x\sqrt{x}\right) \cdot (5x^2 + 1) - 40x\sqrt{x} \cdot (3 - 5x^2)\right)}{4 \cdot (5x + 1)^4} & & \\
 &= \frac{\left(\frac{3}{\sqrt{x}} - 25x\sqrt{x}\right) \cdot (5x^2 + 1) - 40x\sqrt{x} \cdot (3 - 5x^2)}{4 \cdot (5x^2 + 1)^3} & & \\
 &= \frac{15x^2 - 125x^4 + 3 - 25x^2 - 120x^2 + 200x^3\sqrt{x}}{4\sqrt{x} \cdot (5x + 1)^3} & & \\
 &= \frac{15x^2 - 125x^4 + 3 - 25x^2 - 120x^2 + 200x^4}{4\sqrt{x} \cdot (5x + 1)^3} & & \\
 &= \frac{75x^4 - 130x^2 + 3}{4\sqrt{x} \cdot (5x + 1)^3}
 \end{aligned}$$

f)  $f(x) = \frac{5\sqrt{x} + 3x}{5x} = \frac{1}{\sqrt{x}} + \frac{3}{5}$

$$\begin{aligned}
 f'(x) &= -\frac{1}{2x\sqrt{x}} \\
 f''(x) &= \frac{3}{4x^2\sqrt{x}}
 \end{aligned}$$

### Lösung A2

a)  $f(x) = \frac{\frac{2}{3}x^3}{5x^2 - x + 7}$

$$\begin{aligned}
 u &= \frac{2}{3}x^3 & u' &= 2x^2 \\
 v &= 5x^2 - x + 7 & v' &= 10x - 1 \\
 f'(x) &= \frac{2x^2 \cdot (5x^2 - x + 7) - \frac{2}{3}x^3 \cdot (10x - 1)}{(5x^2 - x + 7)^2} & & \\
 &= \frac{10x^4 - 2x^3 + 14x^2 - \frac{20}{3}x^4 + \frac{2}{3}x^3}{(5x^2 - x + 7)^2} & & \\
 &= \frac{\frac{10}{3}x^4 - \frac{4}{3}x^3 + 14x^2}{(5x^2 - x + 7)^2} & & \\
 &= \frac{10x^4 - 4x^3 + 42x^2}{3 \cdot (5x^2 - x + 7)^2}
 \end{aligned}$$

b)  $f(x) = \frac{\frac{2}{x^2 - 1}}{x + 5}$

$$\begin{aligned}
 u &= \frac{2}{x^2 - 1} & u' &= -\frac{4}{x^3} \\
 v &= x + 5 & v' &= 1 \\
 f'(x) &= \frac{-\frac{4}{x^3} \cdot (x + 5) - \frac{2}{x^2 - 1}}{(x + 5)^2} & & \\
 &= \frac{-\frac{4}{x^2} \cdot \frac{20}{x^3} - \frac{2}{x^2 - 1}}{(x + 5)^2} & & \\
 &= \frac{-6x - 20 + x^3}{x^3 \cdot (x + 5)^2} & & \\
 f''(x) &= \frac{60}{x^4} + \frac{4}{x^3}
 \end{aligned}$$

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c)  $f(x) = \frac{5\sqrt{x}+3x}{5x+7}$        $u = 5\sqrt{x} + 3x$        $u' = \frac{5}{2\sqrt{x}} + 3$   
 $v = 5x + 7$        $v' = 5$

$$f'(x) = \frac{\left(\frac{5}{2\sqrt{x}}+3\right)\cdot(5x+7)-5\cdot(5\sqrt{x}+3x)}{(5x+7)^2} = \frac{\frac{25x}{2\sqrt{x}}+15x+\frac{35}{2\sqrt{x}}+21-25\sqrt{x}-15x}{(5x+7)^2}$$

$$= \frac{\frac{25x+35+42\sqrt{x}-50x}{2\sqrt{x}}}{(5x+7)^2} = \frac{42\sqrt{x}-25x+35}{2\sqrt{x}\cdot(5x+7)^2}$$

d)  $f(x) = \frac{-\frac{1}{x^2} + \frac{1}{10}x}{\sqrt{x}}$        $u = -\frac{1}{x^2} + \frac{1}{10}x$        $u' = \frac{2}{x^3} + \frac{1}{10}$   
 $v = \sqrt{x}$        $v' = \frac{1}{2\sqrt{x}}$

$$f'(x) = \frac{\left(\frac{2}{x^3}+\frac{1}{10}\right)\cdot\sqrt{x}-\left(-\frac{1}{x^2}+\frac{1}{10}x\right)\frac{1}{2\sqrt{x}}}{x} = \frac{2x\cdot\left(\frac{2}{x^3}+\frac{1}{10}\right)+\frac{1}{x^2}-\frac{1}{10}x}{2x\sqrt{x}}$$

$$= \frac{\frac{4}{x^2}+\frac{1}{10}x+\frac{1}{x^2}}{2x\sqrt{x}} = \frac{40+x^3+10}{20x^3\sqrt{x}} = \frac{x^3+50}{20x^3\sqrt{x}}$$

e)  $f(x) = \frac{7x^5+x^2-2x}{x^7+3x}$        $u = 7x^5 + x^2 - 2x$        $u' = 35x^4 + 2x - 2$   
 $v = x^7 + 3x$        $v' = 7x^6 + 3$

$$f'(x) = \frac{(35x^4+2x-2)\cdot(x^7+3x)-(7x^5+x^2-2x)\cdot(7x^6+3)}{(x^7+3x)^2}$$

$$= \frac{35x^{11}+105x^5+2x^8+6x^2-2x^7-6x-(49x^{11}+7x^8-14x^7+21x^5+3x^2-6x)}{(x^7+3x)^2}$$

$$f'(x) = -\frac{14x^{11}+5x^8-12x^7-84x^5-3x^2}{(x^7+3x)^2} = -\frac{14x^9+5x^6-12x^5-84x^3-3}{(x^6+3)^2}$$

f)  $f_a(t) = \frac{\sin(at)+at}{t^2}$        $u = \sin(at) + at$        $u' = a \cdot \cos(at) + a$   
 $v = t^2$        $v' = 2t$

$$f'_a(t) = \frac{(a \cdot \cos(at)+a) \cdot t^2 - 2t \cdot (\sin(at)+at)}{t^4} = \frac{t \cdot (at \cdot \cos(at)+at-2 \sin(at)-2at)}{t^4}$$

$$= \frac{at \cdot \cos(at)-2 \sin(at)-at}{t^3}$$

### Lösung A3

a)  $f_1(x) = \frac{\frac{1}{x}+1}{3x+5}$        $u = \frac{1}{x} + 1$        $u' = -\frac{1}{x^2}$   
 $v = 3x + 5$        $v' = 3$

$$f'_1(x) = \frac{-\frac{1}{x^2}(3x+5)-3 \cdot \frac{1}{x}+1}{(3x+5)^2}$$

$$f'_1(-1) = -\frac{1}{2}$$

b)  $f_2(x) = \frac{2x^6}{\frac{1}{x^3}-\frac{1}{x^4}}$        $u = 2x^6$        $u' = 12x^5$   
 $v = \frac{1}{x^3}-\frac{1}{x^4}$        $v' = -\frac{3}{x^4}+\frac{4}{x^5}$

$$f'_2(x) = \frac{12x^5 \cdot \left(\frac{1}{x^3}-\frac{1}{x^4}\right)-2x^6 \cdot \left(-\frac{3}{x^4}+\frac{4}{x^5}\right)}{\left(\frac{1}{x^3}-\frac{1}{x^4}\right)^2}$$

$$f'_2(2) = 8192$$

c)  $f_3(x) = \frac{\sqrt{x}}{5x^2+\frac{1}{x^2}}$        $u = \sqrt{x}$        $u' = \frac{1}{2\sqrt{x}}$   
 $v = 5x^2 + \frac{1}{x^2}$        $v' = 10x - \frac{2}{x^3}$

$$f'_3(x) = \frac{\frac{1}{2\sqrt{x}}\left(5x^2+\frac{1}{x^2}\right)-\sqrt{x} \cdot \left(10x-\frac{2}{x^3}\right)}{\left(5x^2+\frac{1}{x^2}\right)^2}$$

$$f'_3(1) = -\frac{5}{36}$$

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d)  $f_4(x) = \frac{\frac{1}{8}x^3 + \frac{6}{5}x}{\frac{2}{5}x - 8}$

$$f'_4(x) = \frac{\left(\frac{3}{8}x^2 + \frac{6}{5}\right) \cdot \left(\frac{2}{5}x - 8\right) - \frac{2}{5} \left(\frac{1}{8}x^3 + \frac{6}{5}x\right)}{\left(\frac{2}{5}x - 8\right)^2}$$

$$f'_4(0) = -\frac{3}{20}$$

$$u = \frac{1}{8}x^3 + \frac{6}{5}x \quad u' = \frac{3}{8}x^2 + \frac{6}{5}$$

$$v = \frac{2}{5}x - 8 \quad v' = \frac{2}{5}$$

### Lösung A4

a)  $f(x) = \frac{x+3x^2}{2x-1}$

$$f'(x) = \frac{(1+6x) \cdot (2x-1) - 2 \cdot (x+3x^2)}{(2x-1)^2} = \frac{6x^2 - 6x - 1}{(2x-1)^2}$$

$$g(x) = \frac{1}{(2x-1)}$$

$$g'(x) = -\frac{2}{(2x-1)^2}$$

$$f'(x) \cap g'(x)$$

$$\frac{6x^2 - 6x - 1}{(2x-1)^2} = -\frac{2}{(2x-1)^2}$$

$$6x^2 - 6x + 1 = 0 \quad | \quad :6$$

$$x^2 - x + \frac{1}{6} = 0$$

$$x_{1,2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{1}{6}} = -\frac{1}{2} \pm \sqrt{\frac{1}{12}} = -\frac{1}{2} \pm \frac{1}{2} \cdot \sqrt{\frac{1}{3}} = -\frac{1}{2} \pm \frac{1}{6}\sqrt{3}$$

$$m_1 = g'\left(-\frac{1}{2} + \frac{1}{6}\sqrt{3}\right) \approx -1 \quad m_2 = g'\left(-\frac{1}{2} - \frac{1}{6}\sqrt{3}\right) \approx -0,3$$

An den Stellen  $x_1 = -\frac{1}{2} + \frac{1}{6}\sqrt{3}$  und  $x_2 = -\frac{1}{2} - \frac{1}{6}\sqrt{3}$  verlaufen die beiden Graphen mit einer Steigung von  $m_1 \approx -1$  und  $m_2 \approx -0,3$  parallel.

b)  $f(x) = \frac{\frac{1}{2}x^2 + 8x}{x^2 - 4x}$

$$f'(x) = \frac{(x+8) \cdot (x^2 - 4x) - \left(\frac{1}{2}x^2 + 8x\right) \cdot (2x-4)}{(x^2 - 4x)^2} = \frac{10}{(x-4)^2}$$

$$g(x) = 3x + 2$$

$$g'(x) = 3$$

$$f'(x) \cap g'(x)$$

$$\frac{10}{(x-4)^2} = 3$$

$$(x-4)^2 = \frac{10}{3}$$

$$|x-4| = \sqrt{\frac{10}{3}}$$

$$x_1 = 4 + \sqrt{\frac{10}{3}}; \quad x_2 = 4 - \sqrt{\frac{10}{3}}$$

An den Stellen  $x_1 = 4 + \sqrt{\frac{10}{3}}$  und  $x_2 = 4 - \sqrt{\frac{10}{3}}$  verlaufen die beiden Graphen mit einer Steigung von  $m = 3$  parallel.