Lösung A1

a) $f(x) = \frac{x}{sin(3x)}$ $\quad u = x \quad u' = 1$

$v = sin(3x) \quad v' = 3cos(3x)$

$f'(x) = \frac{\sin(3x) - 3x cos(3x)}{(\sin(3x))^2}$

b) $f(x) = \frac{(3x+4)^2}{sin(x)}$ $\quad u = (3x+4)^2 \quad u' = 6(3x+4)$

$v = sin(x) \quad v' = cos(x)$

$f'(x) = \frac{6(3x+4)\, sin(x) - (3x+4)^2 cos(x)}{sin^2(x)}$

$= \frac{(3x+4)\cdot(6\cdot sin(x) - 3x\cdot cos(x) - 4\, cos(x))}{sin^2(x)}$

c) $f(x) = \frac{x^{-1}}{2x+3}$ $\quad u = x^{-1} \quad u' = -x^{-2}$

$v = (2x+3) \quad v' = 2$

$f'(x) = \frac{-x^{-2}\cdot(2x+3) - 2x^{-1}}{(2x+3)^2} = \frac{-\frac{2x+3}{x^2} - \frac{2}{x}}{(2x+3)^2} = \frac{-2x-3-2x}{x^2\cdot(2x+3)^2} = -\frac{4x+3}{x^2\cdot(2x+3)^2}$

d) $f(x)\frac{(5-4x)^3}{1-4x}$ $\quad u = (5-4x)^3 \quad u' = -12(5-4x)^2$

$v = 1-4x \quad v' = -4$

$f'(x) = \frac{-12(5-4x)^2\cdot(1-4x) + 4\cdot(5-4x)^3}{(1-4x)^2} = \frac{4(5-4x)^2\cdot\left(1-3(5-4x)\cdot(1-4x)\right)}{(1-4x)^2}$

$= \frac{4(5-4x)^3 - 12(1-4x)(5-4x)^2}{(1-4x)^2} = \frac{4(5-4x)^2\cdot\left(5-4x-3\cdot(1-4x)\right)}{(1-4x)^2}$

$= \frac{4(5-4x)^2\cdot(2+8x)}{(1-4x)^2} = \frac{8(5-4x)^2\cdot(4x+1)}{(1-4x)^2}$

e) $f(x) = \frac{(5-4x)^{-3}}{x^{-2}} = \frac{x^2}{(5-4x)^3}$ $\quad u = x^2 \quad u' = 2x$

$v = (5-4x)^3 \quad v' = -12(5-4x)^2$

$f'(x) = \frac{2x\cdot(5-4x)^3 + 12x^2(5-4x)^2}{(5-4x)^6} = \frac{(5-4x)^2\cdot\left(2x\cdot(5-4x) + 12x^2\right)}{(5-4x)^6}$

$= \frac{10x - 8x^2 + 12x^2}{(5-4x)^4} = \frac{4x^2+10x}{(5-4x)^4}$

f) $f(x) = \frac{3x}{cos(2x)}$ $\quad u = 3x \quad u' = 3$

$v = cos(2x) \quad v' = -2sin(2x)$

$f'(x) = \frac{3\cos(2x) + 6x sin(2x)}{(\cos(2x))^2}$

g) $f(x) = \frac{3x}{(sin(x))^2}$ $\quad u = 3x \quad u' = 3$

$v = (sin(x))^2 \quad v' = 2\cdot sin(x)\cdot cos(x)$

$f'(x) = \frac{3(sin(x))^2 - 6x sin(x)\, cos(x)}{(sin(x))^4} = \frac{3sin(x)\cdot\left(sin(x) - 2x cos(x)\right)}{(sin(x))^4}$

$= \frac{3\left(sin(x) - 2x cos(x)\right)}{(sin(x))^3}$

h) $f(x) = \frac{0{,}5x^2}{\sqrt{4-x}}$ $\quad u = 0{,}5x^2 \quad u' = x$

$v = \sqrt{4-x} \quad v' = -\frac{1}{2\sqrt{4-x}}$

$$f'(x) = \frac{x\sqrt{4-x}+\frac{0{,}5x^2}{2\sqrt{4-x}}}{4-x} = \frac{2x(4-x)+0{,}5x^2}{2(4-x)\cdot\sqrt{4-x}} = -\frac{\frac{3}{2}x^2-8x}{2(4-x)^{\frac{3}{2}}}$$
$$= -\frac{3x^2-16x}{4(4-x)^{\frac{3}{2}}}$$

Lösung A2

a) Es wurde nur $-v' \cdot u$ gebildet und die Multiplikation $u' \cdot v$ wurde vergessen.

b) $g'(x) = \frac{2\cdot(8-x)^2+2\cdot(8-x)\cdot(2x-3)}{(8-x)^2}$

Lösung A3

$f(x) = f(x) = \frac{x-1}{\sqrt{x}}$

a) Schnittpunkte des Graphen mit der x-Achse mithilfe von $f(x) = 0$:

$\frac{x-1}{\sqrt{x}} = 0$

$x_0 = 1$

b) Steigung der Tangente in $P(1|f(1))$:

$f(x) = \frac{x-1}{\sqrt{x}}$ $\quad u = x-1 \quad u' = 1$

$v = \sqrt{x} \quad v' = \frac{1}{2\sqrt{x}}$

$f'(x) = \frac{\sqrt{x}-\frac{1}{2\sqrt{x}}(x-1)}{x}$

$f'(1) = 1$

Die Tangente in $P(1|f(1))$ hat die Steigung $m = 1$.

c) Waagrechte Tangenten mit $f'(x) = 0$:

$\frac{\sqrt{x}-\frac{1}{2\sqrt{x}}(x-1)}{x} = 0 \quad | \cdot x$

$\sqrt{x} - \frac{1}{2\sqrt{x}}(x-1) = 0 \quad | \cdot 2\sqrt{x}$

$2x - x + 1 = 0$

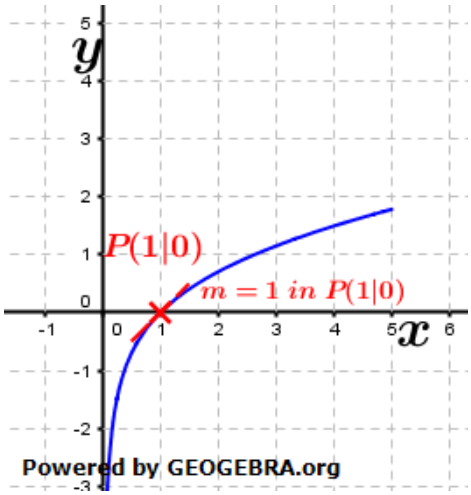
$x + 1 = 0$

$x_0 = -1$

$f(-1)$ hat keine Lösung, da $\sqrt{-1}$ in $\mathbb{R}$ nicht existiert

Der Graph von f hat keine waagrechte Tangente.

d)



Lösung A4

a) $f(x) = \frac{2x-3}{cos(x)}$

$f'(x) = \frac{2\,cos(x)+(2x-3)sin(x)}{cos^2(x)}$

$h(x) = \frac{(2x-3)^3}{3x}$

$h'(x) = \frac{(2x-3)^2\cdot(4x+3)}{3x^2}$

$g(x) = \frac{x}{(1-x)^2}$

$g'(x) = -\frac{x+1}{(x-1)^3}$

$i(x) = \frac{1}{x\cdot sin(x)}$

$i'(x) = \frac{sin(x)+xcos(x)}{x^2\cdot sin^2(x)}$

b) $g'(x) = 0$

$-\frac{x+1}{(x-1)^3} = 0$

$x_1 = -1$

$g(-1) = -\frac{1}{4}$

Der Graph von g hat in $P(-1|-\frac{1}{4})$ eine waagrechte Tangente.

c) $h(x) = 0$

$\frac{(2x-3)^3}{3x} = 0 \quad | \quad \cdot 3x$

$(2x-3)^3 = 0 \quad | \quad \sqrt[3]{}$

$2x-3 = 0$

$x = \frac{3}{2}$

Der Graph von h schneidet die x-Achse in $x_0 = \frac{3}{2}$.

d) $h'\left(\frac{3}{2}\right)$

$h'\left(\frac{3}{2}\right) = 0$

Lösung A5

$f(x) = \frac{x^2}{g(x)}$

$f'(x) = \frac{2x\cdot g(x)-x^2\cdot g'(x)}{(g(x))^2}$

$f''(x) = \frac{2\cdot g(x)+2xg'(x)-2x\cdot g'(x)-x^2g''(x)}{(g(x))^4}$

$= \frac{2\cdot g(x)-x^2g''(x)}{(g(x))^4}$

$f(x) = \frac{x}{g'(x)}$

$f'(x) = \frac{-x\cdot g''(x)}{(g'(x))^2}$

$f''(x) = \frac{-g''(x)-xg'''(x)}{(g'(x))^4}$

$f(x) = \frac{g(x)}{g'(x)}$

$f'(x) = \frac{(g'(x))^2-g(x)\cdot g''(x)}{(g'(x))^2} = 1-\frac{g(x)\cdot g''(x)}{(g'(x))^2}$

$f''(x) = -\frac{g'(x)\cdot g''(x)+g(x)\cdot g'''(x)}{(g'(x))^4}$