

# Aufgabenblatt Ableitungen

Differenzialrechnung

zur Summen- bzw. Differenzregel

Lösungen

Level 4 – Universität – Blatt 1

## Lösung A1

Differenzenquotient:

$$\frac{\Delta y}{\Delta x} = \frac{g(x+h) - g(x) + k(x+h) - k(x)}{x+h-x} = \frac{3(x+h)^2 - 3x^2 + (x+h+2)^3 - (x+2)^3}{h}$$

Daraus folgt:

$$\begin{aligned}\frac{\Delta y}{\Delta x} &= \frac{3x^2 + 6hx + 3h^2 - 3x^2 + (x+h+2)^2 \cdot (x+h+2) - (x^3 + 6x^2 + 12x + 8)}{h} \\ &= \frac{6hx + 3h^2 + (x^2 + 2hx + 4x + 4h + h^2 + 4)(x+h+2) - x^3 - 6x^2 - 12x - 8}{h} \\ &= \frac{h(6x + 3h) + x^3 + 3hx^2 + 6x^2 + 3h^2x + 12hx + 12x + h^3 + 6h^2 + 12h + 8 - x^3 - 6x^2 - 12x - 8}{h} \\ &= \frac{h(6x + 3h) + 3hx^2 + 3h^2x + 12hx + 12x + h^3 + 6h^2 + 12h}{h} \\ &= \frac{h(6x + 3h) + h(3x^2 + 3hx + 12x + h^2 + 6h + 12)}{h} \\ &= (6x + 3h) + (3x^2 + 3hx + 12x + h^2 + 6h + 12) \\ \frac{dy}{dx} &= f'(x) = \lim_{h \rightarrow 0} (6x + 3h) + (3x^2 + 3hx + 12x + h^2 + 6h + 12) = (6x) + (3x^2 + 12x + 12) \\ g(x) &= 3x^2; & g'(x) &= 6x \\ k(x) &= (x+2)^3 = x^3 + 6x^2 + 12x + 8 & k'(x) &= 3x^2 + 12x + 12 \\ f'(x) &= g'(x) + k'(x) = 6x + 3x^2 + 12x + 12\end{aligned}$$

q.e.d.