Lösung A1

Detaillierte Lösung für a)

Bestimmung von $f'(x)$ $\qquad f'(x) = 3 - 4x$

Bestimmung von $f(u)$ für $u = -2$ $\qquad f(-2) = 3 \cdot (-2) - 2 \cdot (-2)^2 = -14$

Bestimmung von $f'(u)$ für $u = -2$ $\qquad f'(-2) = 3 - 4 \cdot (-2) = 11$

Aufstellung der Tangentengleichung

$t(x) = f'(u) \cdot (x - u) + f(u)$ $\qquad t(x) = 11 \cdot (x + 2) - 14$

$t(x) = 11x + 8$

Aufstellung der Normalengleichung

$n(x) = -\frac{1}{f'(u)} \cdot (x - u) + f(u)$ $\qquad n(x) = -\frac{1}{11} \cdot (x + 2) - 14$

$n(x) = -\frac{1}{11}x - \frac{156}{11}$

b) $f(x) = x^3 - x^2; \quad u = 4$ $\qquad f'(x) = 3x^2 - 2x$

$f'(4) = 3 \cdot 16 - 8 = 40$ $\qquad f(4) = 4^3 - 4^2 = 48$

$t(x) = 40(x - 4) + 48$ $\qquad n(x) = -\frac{1}{40}(x - 4) + 48$

$t(x) = 40x - 112$ $\qquad n(x) = -\frac{1}{40}x + \frac{481}{10}$

c) $f(x) = -\frac{1}{4}x^2(x - 6); \quad u = 1$ $\qquad f'(x) = -\frac{3}{4}x^2 + 3x$

$f'(1) = -\frac{3}{4} + 3 = \frac{9}{4}$ $\qquad f(1) = \frac{5}{4}$

$t(x) = \frac{9}{4}(x - 1) + \frac{5}{4}$ $\qquad n(x) = -\frac{4}{9}(x - 1) + \frac{5}{4}$

$t(x) = \frac{9}{4}x - 1$ $\qquad n(x) = -\frac{4}{9}x + \frac{61}{36}$

d) $f(x) = \frac{1}{4}x^4 - \frac{3}{2}x^2; \quad u = \sqrt{3}$ $\qquad f'(x) = x^3 - 3x$

$f'(\sqrt{3}) = 0$ $\qquad f(\sqrt{3}) = \frac{9}{4} - \frac{9}{2} = -\frac{9}{4}$

$t(x) = -\frac{9}{4}$

Die Normale ist eine Parallele zur y-Achse selbst im Abstand $\sqrt{3}$

$x = \sqrt{3}$

Lösung A2

$f(x) = (x - 2)^2 + x$ $\qquad f'(x) = 2(x - 2) + 1 = 2x - 3$

a) I) $f'(0) = -3$ $\quad$ II) $f'(1) = -1$ $\quad$ III) $f'(1{,}5) = 0$ $\quad$ IV) $f'(2) = 1$

V) $f'(-2) = -7$

b) I) $f(0) = 4$ $\qquad t(x) = -3x + 4$

II) $f(1) = 2$ $\qquad t(x) = -(x - 1) + 2$

$t(x) = -x + 3$

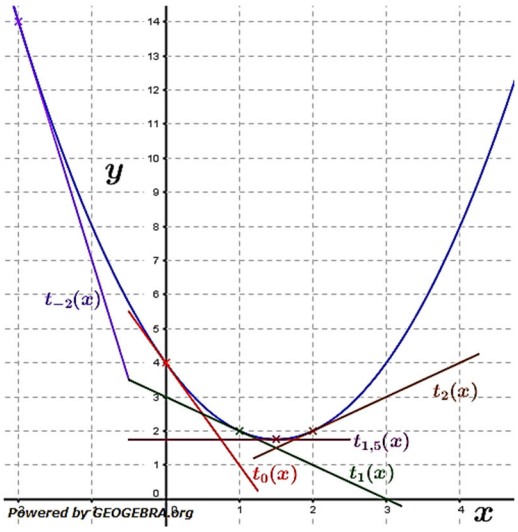
III) $f(1{,}5) = 1{,}75$ $\qquad t(x) = 1{,}75$

IV) $f(2) = 2$ $\qquad t(x) = (x - 2) + 2$

$t(x) = x$

V) $f(-2) = 14$ $\qquad t(x) = -7(x + 1) + 14$

$t(x) = -7x + 7$



Lösung A3

a) $f(x) = \frac{1}{4}x^2(x-3) = \frac{1}{4}x^3 - \frac{3}{4}x^2$ $\qquad f'(x) = \frac{3}{4}x^2 - \frac{3}{2}x$

$f(1) = -\frac{1}{2}$ $\qquad f'(1) = -\frac{3}{4}$

$t(x) = -\frac{3}{4}(x-1) - \frac{1}{2}$ $\qquad n(x) = \frac{4}{3}(x-1) - \frac{1}{2}$

$t(x) = -\frac{3}{4}x + \frac{1}{4}$ $\qquad n(x) = \frac{4}{3}x - \frac{11}{6}$

b) $f'(x) = 2{,}25 = \frac{3}{4}x^2 - \frac{3}{2}x$

$\frac{3}{4}x^2 - \frac{3}{2}x - \frac{9}{4} = 0$ $\quad | \quad \cdot\frac{4}{3}$

$x^2 - 2x - 3 = 0$

$x_{1,2} = 1 \pm \sqrt{1+3} = 1 \pm 2$ $\quad | \quad p/q$-Formel

$x_1 = 3; \quad x_2 = -1$

$f(3) = \frac{27}{4} - \frac{27}{4} = 0$ $\qquad f(-1) = -\frac{1}{4} - \frac{3}{4} = -1$

$t_1(x) = 2{,}25(x-3) + 0$ $\qquad t_2(x) = 2{,}25(x-1) - 1$

$t_1(x) = 2{,}25x - 6{,}75$ $\qquad t_2(x) = 2{,}25x - 3{,}25$

c) $m_1 \cdot m_2 = -1$

$1{,}5 \cdot m_2 = -1 \implies m_2 = -\frac{2}{3}$

$f'(x) = -\frac{2}{3} = \frac{3}{4}x^2 - \frac{3}{2}x$

$\frac{3}{4}x^2 - \frac{3}{2}x + \frac{2}{3} = 0$ $\quad | \quad \cdot\frac{4}{3}$

$x^2 - 2x + \frac{8}{9} = 0$

$x_{1,2} = 1 \pm \sqrt{1 - \frac{8}{9}} = 1 \pm \frac{1}{3}$ $\quad | \quad p/q$-Formel

$x_1 = \frac{4}{3}; \quad x_2 = \frac{2}{3}$

$f\left(\frac{4}{3}\right) = \frac{16}{27} - \frac{4}{3} = -\frac{20}{27}$ $\qquad f\left(\frac{2}{3}\right) = \frac{2}{27} - \frac{1}{3} = -\frac{7}{27}$

$t_1(x) = -\frac{2}{3}\left(x - \frac{4}{3}\right) - \frac{20}{27}$ $\qquad t_2(x) = -\frac{2}{3}\left(x - \frac{2}{3}\right) - \frac{7}{27}$

$t_1(x) = -\frac{2}{3}x + \frac{4}{27}$ $\qquad t_2(x) = -\frac{2}{3}x + \frac{5}{27}$

d) Waagrechte Tangente bedeutet $f'(x) = 0$.

$\frac{3}{4}x^2 - \frac{3}{2}x = 0$

$x \cdot \left(\frac{3}{4}x - \frac{3}{2}\right) = 0$ $\quad |$ Satz vom Nullprodukt

$x_1 = 0; \quad x_2 = 2$

$f(0) = 0; \quad f(2) = -1$

In den Punkten $P(0|0)$ und $Q(2|-1)$ besitzt K eine waagrechte Tangente.

e) $m_1 \cdot m_2 = -1$

$2{,}4 \cdot m_2 = -1 \implies m_2 = -\frac{5}{12}$

$f'(x) = -\frac{5}{12} = \frac{3}{4}x^2 - \frac{3}{2}x$

$\frac{3}{4}x^2 - \frac{3}{2}x + \frac{5}{12} = 0 \qquad | \quad \cdot\frac{4}{3}$

$x^2 - 2x + \frac{5}{9} = 0$

$x_{1,2} = 1 \pm \sqrt{1 - \frac{5}{9}} = 1 \pm \frac{2}{3} \qquad | \quad p/q$-Formel

$x_1 = \frac{5}{3}; \quad x_2 = \frac{1}{3}$

$f\left(\frac{5}{3}\right) = -\frac{25}{27} \qquad f\left(\frac{1}{3}\right) = -\frac{2}{27}$

In den Punkten $P\left(\frac{5}{3}\middle|-\frac{25}{27}\right)$ und $Q\left(\frac{1}{3}\middle|-\frac{2}{27}\right)$ verläuft die Normale von K parallel zur Ursprungsgeraden mit der Steigung $m = 24$.