

### Lösung A1

Detaillierte Lösung für a)

Funktion - Berührungspunkt

$$f(x) = \frac{x+1}{x}; \quad P(1|f(1))$$

Bestimmung von  $f'(x)$  (Quotientenregel)

$$f'(x) = \frac{x-(x+1)}{x^2} = -\frac{1}{x^2}$$

Bestimmung von  $f(1)$

$$f(1) = \frac{1+1}{1} = 2$$

Bestimmung von  $f'(1)$

$$f'(1) = -\frac{1}{1} = -1$$

Aufstellung der Tangentengleichung

$$t(x) = f'(1) \cdot (x - 1) + f(1)$$

$$t(x) = -(x - 1) + 2$$

$$t(x) = -x + 3$$

b)  $f(x) = \frac{x^2}{x+1}; \quad P(1|f(1))$

$$f'(x) = \frac{2x(x+1)-2x}{(x+1)^2} = \frac{2x^2}{(x+1)^2}$$

$$f(1) = \frac{1}{2}$$

$$f'(1) = \frac{1}{2}$$

$$t(x) = \frac{1}{2}(x - 1) + \frac{1}{2}$$

$$t(x) = \frac{1}{2}x$$

c)  $f(x) = \frac{x^2}{(x+2)^2}; \quad P(2|f(2))$

$$f'(x) = \frac{2x(x+2)^2 - x^2 \cdot 2 \cdot (x+2)}{(x+2)^4} = \frac{4x}{(x+2)^3}$$

$$f(2) = \frac{1}{4}$$

$$f'(2) = \frac{1}{8}$$

$$t(x) = \frac{1}{8}(x - 2) + \frac{1}{4}$$

$$t(x) = \frac{1}{8}x$$

### Lösung A2

Detaillierte Lösung für a)

Funktion - Steigung

$$f(x) = 2x^3; \quad m_t = 6$$

Bestimmung von  $f'(x)$

$$f'(x) = 6x$$

Gleichsetzung  $f'(x) = m_t$

$$6x = 6$$

$$| \quad :6$$

$$x = 1$$

Es gibt nur eine einzige Tangente.

Bestimmung von  $f(1)$

$$f(1) = 2$$

Aufstellung der Tangentengleichung

$$t(x) = f'(1) \cdot (x - 1) + f(1)$$

$$t(x) = 6(x - 1) + 2$$

$$t(x) = 6x - 4$$

b)  $f(x) = \frac{1}{4}x^2 + x; \quad m_t = \frac{1}{2}$

$$f'(x) = \frac{1}{2}x + 1$$

$$\frac{1}{2}x + 1 = \frac{1}{2}$$

$$x = -1$$

$$f(-1) = \frac{1}{4} - 1 = -\frac{3}{4}$$

$$f'(-1) = \frac{1}{2}$$

$$t(x) = \frac{1}{2}(x + 1) - \frac{3}{4}$$

$$t(x) = \frac{1}{2}x - \frac{1}{4}$$

c)  $f(x) = \frac{x}{x+1}; \quad m_t = 4$

$$f'(x) = \frac{x+1-x}{(x+1)^2} = \frac{1}{(x+1)^2}$$

$$\frac{1}{(x+1)^2} = 4$$

$$(x+1)^2 = \frac{1}{4}$$

$$| \quad \sqrt{\quad}$$

$$|x+1| = \frac{1}{2}$$

$$x_1 = -\frac{1}{2}; \quad x_2 = -\frac{3}{2}$$

Es gibt zwei Tangenten.

Level 2 - Fortgeschritten - Blatt 2

$$f\left(-\frac{1}{2}\right) = \frac{-0,5}{-0,5+1} = -1$$

$$t_1(x) = 4\left(x + \frac{1}{2}\right) - 1$$

$$t_1(x) = 4x + 1$$

d)  $f(x) = \frac{1}{2}\sqrt{2x+1}; \quad m_t = 1$

$$\frac{1}{2\sqrt{2x+1}} = 1$$

$$2\sqrt{2x+1} = 1$$

$$4(2x+1) = 1$$

$$8x + 4 = 1$$

$$8x = -3$$

$$x = -\frac{3}{8}$$

$$f\left(-\frac{3}{8}\right) = \frac{1}{2}\sqrt{-\frac{6}{8}+1} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$t(x) = \left(x + \frac{3}{8}\right) + \frac{1}{4}$$

$$t(x) = x + \frac{5}{8}$$

e)  $f(x) = \sqrt{x} + x; \quad m_t = 2$

$$\frac{1}{2\sqrt{x}} + 1 = 2$$

$$2\sqrt{x} = 1$$

$$4x = 1$$

$$x = \frac{1}{4}$$

$$f\left(\frac{1}{4}\right) = \sqrt{\frac{1}{4}} + \frac{1}{4} = \frac{3}{4}$$

$$t(x) = 2\left(x - \frac{1}{4}\right) + \frac{3}{4}$$

$$t(x) = 2x + \frac{1}{4}$$

$$f\left(-\frac{3}{2}\right) = \frac{-1,5}{-1,5+1} = 3$$

$$t_2(x) = 4\left(x + \frac{3}{2}\right) + 3$$

$$t_2(x) = 4x + 9$$

$$f'(x) = \frac{1}{2} \cdot \frac{1}{2}(2x+1)^{-\frac{1}{2}} \cdot 2 = \frac{1}{2\sqrt{2x+1}}$$

|      2

$$f'(x) = \frac{1}{2\sqrt{x}} + 1$$

|      2

## Lösung A3

Detaillierte Lösung für a)

Funktion

Steigung Gerade  $g(x) = 7x + 4$

Steigung der Normalen ( $m_g \cdot m_n = -1$ )

Bestimmung von  $f'(x)$

Gleichsetzung  $f'(x) = m_g$

$$3x + 1 = 7$$

$$x = 2$$

Bestimmung von  $f(2)$

Aufstellung der Normalengleichung

$$n(x) = -\frac{1}{7} \cdot (x - 2) + 6$$

$$n(x) = -\frac{1}{7}x + \frac{44}{7}$$

$$h(x) = 1,5x^2 + x - 2$$

$$m_g = 7$$

$$m_N = -\frac{1}{7}$$

$$h'(x) = 3x + 1$$

$$-1; : 3$$

$$h(2) = 1,5 \cdot (2)^2 + 2 - 2 = 6$$

Level 2 – Fortgeschritten – Blatt 2

b)  $h(x) = 2x^2 + 5x - 3$

$$m_g = 3$$

$$h'(x) = 4x + 5$$

$$4x + 5 = 3$$

$$x = -\frac{1}{2}$$

$$h\left(-\frac{1}{2}\right) = 2 \cdot \left(-\frac{1}{2}\right)^2 + 5 \cdot \left(-\frac{1}{2}\right) - 3 = -5$$

$$n(x) = -\frac{1}{3}\left(x + \frac{1}{2}\right) - 5$$

$$n(x) = -\frac{1}{3}x - \frac{31}{6}$$

$$g(x) = 3x - 2$$

$$m_N = -\frac{1}{3}$$

$$-5; : 4$$

c)  $h(x) = \frac{3}{4}x^2 - 3,5x + 6$

$$m_g = -\frac{1}{2}$$

$$h'(x) = \frac{3}{2}x - 3,5$$

$$\frac{3}{2}x - 3,5 = -\frac{1}{2}$$

$$3x = 6$$

$$x = 3$$

$$h(3) = \frac{3}{4} \cdot 2^2 - 3,5 \cdot 2 + 6 = 2$$

$$n(x) = 2(x - 3) + 2$$

$$n(x) = 2x - 4$$

$$g(x) = -\frac{1}{2}x - 11$$

$$m_N = 2$$

$$+3,5; \cdot 2$$

d)  $h(x) = x^2 - x + 1$

$$m_g = \frac{3}{2}$$

$$h'(x) = 2x - 1$$

$$2x - 1 = \frac{3}{2}$$

$$x = \frac{5}{4}$$

$$h\left(\frac{5}{4}\right) = \left(\frac{5}{4}\right)^2 - \frac{5}{4} + 1 = \frac{21}{16}$$

$$n(x) = -\frac{2}{3}\left(x - \frac{5}{4}\right) + \frac{21}{16}$$

$$n(x) = -\frac{2}{3}x + \frac{103}{48}$$

$$g(x) = 1,5x - 2$$

$$m_N = -\frac{2}{3}$$

$$+1; : 2$$

e)  $h(x) = \frac{1}{6}x^3 - \frac{3}{4}x^2 - \frac{7}{4}x + \frac{1}{4}$

$$m_g = \frac{1}{4}$$

$$h'(x) = \frac{1}{2}x^2 - \frac{3}{2}x - \frac{7}{4}$$

$$\frac{1}{2}x^2 - \frac{3}{2}x - \frac{7}{4} = \frac{1}{4}$$

$$x^2 - 3x - 4 = 0$$

$$x_{1,2} = 1,5 \pm \sqrt{2,25 + 4} = 1,5 \pm 2,5$$

$$x_1 = 4; \quad x_2 = -1$$

Es gibt zwei Normalen:

$$h(4) = \frac{32}{6} - 12 - 7 + \frac{1}{4} = -\frac{97}{12}$$

$$n_1(x) = -4(x - 4) - \frac{97}{12}$$

$$n_1(x) = -4x + \frac{95}{12}$$

$$g(x) = \frac{1}{4}x + 1$$

$$m_N = -4$$

$$-\frac{1}{4}; \cdot 2$$

$$h(-1) = -\frac{1}{6} - \frac{3}{4} + \frac{7}{4} + \frac{1}{4} = \frac{13}{12}$$

$$n_2(x) = -4(x + 1) + \frac{13}{12}$$

$$n_2(x) = -4x + \frac{25}{12}$$

## Lösung A4

$$f(x) = x^3 - 3x^2$$

Wendepunkte über  $f''(x) = 0$

$$f'(x) = 3x^2 - 6x$$

$$f''(x) = 6x - 6$$

$$f'''(x) = 6$$

$$f'''(x) = 6 \neq 0$$

$$6x - 6 = 0 \Rightarrow x = 1$$

$$f'''(1) \neq 0$$

|  $x_0 = 1$  ist eine Wendestelle.

$$f(1) = 1^3 - 3 \cdot 1^2 = -2$$

$$f'(1) = 3 \cdot 1^2 - 6 \cdot 1 = -3 = m_t$$

$$m_n = -\frac{1}{m_t} = \frac{1}{3}$$

$$t(x) = -3(x - 1) - 2$$

$$n(x) = \frac{1}{3}(x - 1) - 2$$

$$t(x) = -3x + 1$$

$$n(x) = \frac{1}{3}x - \frac{7}{3}$$