

# Differenzialrechnung

## Aufgabenblatt Ableitungen der trigonometrischen Funktionen

Lösungen  
Level 2 – Fortgeschritten – Blatt 2

### Lösung A1

- a)  $f(x) = -\sin(3x) + \cos(3x)$        $f'(x) = -3\cos(3x) - 3\sin(3x)$   
 b)  $f(x) = 4\cos(2x + 4)$        $f'(x) = -8\sin(2x + 4)$   
 c)  $f(x) = \sin^3(x)$        $f'(x) = 3\sin^2(x) \cdot \cos(x)$   
 d)  $f(x) = \sin(x^3)$        $f'(x) = 3x^2\cos(x^3)$   
 e)  $\cos^4(\sqrt{x})$        $f'(x) = 4\cos^3(\sqrt{x}) \cdot (-\sin(\sqrt{x})) \cdot \frac{1}{2\sqrt{x}}$   

$$= -\frac{2}{\sqrt{x}}\cos^3(\sqrt{x}) \cdot \sin(\sqrt{x})$$
  
 f)  $f(x) = 3\sin^2((x-2)^2)$        $f'(x) = 6\sin((x-2)^2) \cdot \cos((x-2)^2) \cdot (2x-2)$   

$$= 12(x-1)\sin((x-2)^2) \cdot \cos((x-2)^2)$$
  
 g)  $f(x) = \sin(\frac{1}{x} + x^2)$        $f'(x) = (-\frac{1}{x^2} + 2x)(\cos(\frac{1}{x} + x^2))$   
 h)  $f(x) = 2\tan^2(2x)$        $f'(x) = 4 \cdot \tan(2x) \cdot (1 + \tan^2(2x)) \cdot 2$   

$$= 8 \cdot \tan(2x) \cdot (1 + \tan^2(2x))$$

### Lösung A2

- a)  $f(x) = 2\cos(x) - 1; x_0 = \frac{\pi}{4}$        $f'(x) = -2\sin(x)$   
 $f'(\frac{\pi}{4}) = -2\sin(\frac{\pi}{4}) = -2 \cdot \frac{1}{2}\sqrt{2} = -\sqrt{2}$
- b)  $f(x) = 2\sin(3x); x_0 = \frac{2\pi}{3}$        $f'(x) = 6\cos(3x)$   
 $f'(\frac{2\pi}{3}) = 6\cos(2\pi) = 6 \cdot 1 = 6$
- c)  $f(x) = \pi x - \sin(0,5\pi x); x_0 = 2$        $f'(x) = 1 - 0,5\pi\cos(0,5\pi x)$   
 $f'(2) = \pi - 0,5\pi\cos(0,5\pi \cdot 2) = \pi - (-0,5\pi) = 1,5\pi$
- d)  $f(x) = 4\sin(3\pi + 2x) + 1; x_0 = 0$        $f'(x) = 8\cos(3\pi + 2x)$   
 $f'(0) = 8\cos(3\pi) = -8$
- e)  $f(x) = \pi - 2\cos(2(x + \frac{\pi}{3})); x_0 = \frac{5\pi}{6}$        $f'(x) = -4\sin(2(x + \frac{\pi}{3}))$   
 $f'(\frac{5\pi}{6}) = -4\sin(2(\frac{5\pi}{6} + \frac{\pi}{3})) = -4\sin(\frac{7}{3}\pi) = -4 \cdot \frac{1}{2}\sqrt{3} = -2\sqrt{3}$
- f)  $f(x) = 4\cos(\frac{x}{\pi}); x_0 = 0$        $f'(x) = -\frac{4}{\pi}\cos(\frac{x}{\pi})$   
 $f'(0) = -\frac{4}{\pi}\cos(0) = -\frac{4}{\pi}$
- g)  $f(x) = -3\sin(2x^2 - \frac{\pi}{4}); x_0 = \frac{1}{2}\sqrt{\pi}$        $f'(x) = -12x \cdot \cos(2x^2 - \frac{\pi}{4})$   
 $f'(\frac{\pi}{4}) = -12x \cdot \cos(2 \cdot \frac{\pi}{4} - \frac{\pi}{4}) = -6\sqrt{\pi} \cdot \frac{1}{2}\sqrt{2} = -3\sqrt{2\pi}$
- h)  $f(x) = 4\sin^2(5x - \frac{4}{3}\pi); x_0 = \frac{\pi}{3}$        $f'(x) = 20\sin(5x - \frac{4}{3}\pi) \cdot \cos(5x - \frac{4}{3}\pi)$   
 $f'(\frac{\pi}{3}) = 20\sin(\frac{5\pi}{3} - \frac{4}{3}\pi) \cdot \cos(\frac{5\pi}{3} - \frac{4}{3}\pi) = 20 \cdot \frac{1}{2} \cdot \sqrt{3} \cdot \frac{1}{2} = 5\sqrt{3}$
- i)  $f(x) = -t\cos^2(x) + t^2\sin(x); x_0 = \frac{\pi}{2}$        $f'(x) = 2t\cos(x) \cdot \sin(x) + t^2\cos(x)$   
 $f'(\frac{\pi}{2}) = 2t\cos(\frac{\pi}{2}) \cdot \sin(\frac{\pi}{2}) + t^2\cos(\frac{\pi}{2}) = 0$
- j)  $f(x) = \frac{t}{2\sqrt{3}}\cos(x) + t\sin(x); x_0 = \frac{\pi}{3}$        $f'(x) = -\frac{t}{2\sqrt{3}}\sin(x) + t\cos(x)$   
 $f'(\frac{\pi}{3}) = -\frac{t}{2\sqrt{3}}\sin(\frac{\pi}{3}) + t\cos(\frac{\pi}{3}) = -\frac{t}{2\sqrt{3}} \cdot \frac{1}{2}\sqrt{3} + \frac{t}{2} = \frac{t}{4} + \frac{t}{2} = \frac{3}{2}t$

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k)  $f(x) = t^2 \sin(x) + t^2; x_0 = \frac{3\pi}{4}$

$$f'(x) = t^2 \cos(x)$$

$$f'(\frac{3\pi}{4}) = t^2 \cos(\frac{3\pi}{4}) = \frac{1}{2} \sqrt{2} t^2$$

l)  $f(x) = -\frac{t}{2\sqrt{2}} \cos(x) + \frac{t}{\pi x}; x_0 = \frac{\pi}{4}$

$$f'(x) = \frac{t}{2\sqrt{2}} \sin(x) - \frac{t}{\pi x^2}$$

$$f'(\frac{\pi}{2}) = \frac{t}{2\sqrt{2}} \sin(\frac{\pi}{4}) - \frac{t}{\pi^3} = \frac{t}{4} + \frac{16t}{\pi^3} = \frac{t(\pi^3 + 64)}{4\pi^3}$$

### Lösung A3

$$f(x) = \sin^2(x); f'(x) = 2 \sin(x) \cdot \cos(x)$$

1. Winkelhalbierende:  $y = x$  hat die Steigung  $m = 1$ :

$$f'(x) = 1 = 2 \sin(x) \cdot \cos(x) \Rightarrow x_1 = \frac{\pi}{4}; x_2 = \frac{5\pi}{4}$$

$f$  hat in  $x_1 = \frac{\pi}{4}$  sowie  $x_2 = \frac{5\pi}{4}$  dieselbe Steigung wie die erste Winkelhalbierende.

Die  $x$ -Achse selbst hat die Steigung  $m = 1$ :

$$f'(x) = 0 = 2 \sin(x) \cdot \cos(x) \Rightarrow x_1 = 0; x_2 = \frac{\pi}{2}; x_3 = \pi; x_4 = \frac{3}{2}\pi; x_5 = 2\pi$$

$f$  hat in  $x = \{0; \frac{\pi}{2}; \pi; \frac{3}{2}\pi; 2\pi\}$  dieselbe Steigung wie die  $x$ -Achse.

### Lösung A4

$$f(x) = 3 \sin(x) - 1; f'(x) = 3 \cos(x)$$

a)  $3 \cos(x) = 0 \Rightarrow x_1 = \frac{\pi}{2}; x_2 = \frac{3}{2}\pi$

b) Ja, es gibt Stellen mit  $f'(x) > 1$ , da  $\mathbb{W}$  von  $3 \cos(x) = [-3; 3]$