

Aufgabenblatt Ableitungen

vermischte Aufgaben

Differenzialrechnung

Lösungen

Level 3 - Expert - Blatt 1

Lösung A1

a) $f'(x) = \frac{6x}{2+3x^2}$

$$f''(x) = \frac{6(3x^2-2)}{(2+3x^2)^2}$$

b) $f'(x) = \frac{4x+1}{2x^2+x}$

$$f''(x) = -\frac{8x^2+4x+1}{x^2(2x+1)^2}$$

c) $f'(x) = 2\ln(4+x) + \frac{2x}{4+x}$

$$f''(x) = \frac{2(8+x)}{(4+x)^2}$$

d) $f_t'(x) = \frac{2x}{x^2+t}$

$$f_t''(x) = -\frac{2(x^2-t)}{(x^2+t)^2}$$

e) $f'(x) = \frac{3}{(3x+1) \cdot \ln(3x+1)}$

$$f''(x) = -\frac{9(\ln(3x+1)+1)}{(3x+1)^2 \ln^2(3x+1)}$$

Lösung A2

a) $f'(x) = e^x \cdot \cos(e^x + 1)$

$$f''(x) = -e^x(e^x \sin(e^x + 1) - \cos(e^x + 1))$$

b) $f'(x) = e^x \cdot (\sin(x) + \cos(x))$

$$f''(x) = 2e^x \cdot \cos(x)$$

c) $f'(x) = 3e^{3x} \cdot \sin(e^{2x}) + 2e^{5x} \cos(e^{2x})$

$$f''(x) = (9e^{3x} - 4e^{7x}) \sin(e^{2x}) + 16e^{5x} \cos(e^{2x})$$

d) $f'(x) = \frac{3e^{3x}}{2\sqrt{e^{3x}+1}}$

$$f''(x) = \frac{9e^{6x}+18e^{3x}}{4 \cdot (e^{3x}+1) \cdot \sqrt{e^{3x}+1}}$$

e) $f'(x) = -\frac{2x \cdot e^{-x}}{\sqrt{2x+1}}$

$$f''(x) = \frac{(4x^2-2) \cdot e^{-x}}{(2x+1) \cdot \sqrt{2x+1}}$$

f) $f'(x) = \frac{\cos(x)}{\sin(x)}$

$$f''(x) = -\frac{\cos^2(x)}{\sin^2(x)} - 1$$

Lösung A3

a) $f'(x) = \frac{2x+3\cos(x)}{x^2+3\sin(x)}$

$$f''(x) = -\frac{(3x^2-6)\sin(x)+12\cos(x)+2x^2+9}{(3\sin(x)+x^2)^2}$$

b) $f'(x) = \frac{\cos(\ln(3x))}{x}$

$$f''(x) = -\frac{\sin(\ln(3x))+\cos(\ln(3x))}{x^2}$$

c) $f'(x) = \frac{\sqrt{6}}{2x \cdot \sqrt{\ln(x)}}$

$$f''(x) = \frac{\sqrt{6} \cdot (2\ln(x)+1)}{4x^2 \ln(x) \cdot \sqrt{\ln(x)}}$$

d) $f'(x) = \frac{1}{2x}$

$$f''(x) = -\frac{1}{2x^2}$$

e) $f'(x) = \frac{12x+3}{4x^2+2x}$

$$f''(x) = -\frac{3(8x^2+4x+1)}{2x^2(2x+1)^2}$$

Lösung A4

a) $f'(x) = 2x$

$$f''(x) = 2$$

b) $f'(a) = -2a$

$$f''(a) = -2$$

c) $f'(y) = 0$

$$f''(y) = 0$$

d) $f'(x) = -3xe^x \cdot (2x\sin(2x) + (-x-2)\cos(2x))$

$$f''(x) = -3e^x(4x(x+2)\sin(2x) + (3x^2-4x-2)\cos(2x))$$

e) $f(x) = \frac{\sqrt{1-\cos^2(x)}}{\sin(x)} = \frac{\sqrt{\sin^2(x)+\cos^2(x)-\cos^2(x)}}{\sin(x)} = \frac{\sqrt{\sin^2(x)}}{\sin(x)} = 1$

$$f'(x) = 0$$

$$f''(x) = 0$$

f) $f'(x) = \frac{1}{\cos^2(x)} = \tan^2(x) + 1$

$$f''(x) = \frac{2 \cdot \sin(x)}{\cos^3(x)} = 2\tan(x) \cdot \frac{1}{\cos^2(x)}$$

g) $f'(x) = -e^{-x}(\sin(x) - \cos(x))$

$$f''(x) = -2e^{-x} \cdot \cos(x)$$