Aufgabenblatt
zu den Logarithmengesetzen

Level 2 – Fortgeschritten – Blatt 2

Dokument mit 27 Aufgaben

Aufgabe A1

Zerlege die Logarithmen-Terme in Einzelterme. (Alle Buchstaben vertreten positive Zahlen und es ist $a \neq 1$.)

a)	$log_a\left(\sqrt[3]{\frac{2b^3c}{5de^5v}}\right)$	b)	$log_a\left(\frac{3b^2c^4 \cdot \sqrt{d^2}}{\sqrt[3]{2e^2f^3}}\right)$	c)	$log_a\left(\frac{\sqrt[3]{e} \cdot b^2}{5 \cdot \sqrt{3}}\right)$
d)	$log_a\left(\left(\frac{\sqrt[4]{u} \cdot 5^4}{v \cdot \sqrt[3]{12}}\right)^{\frac{1}{5}}\right)$	e)	$log_a\left(\sqrt[3]{\frac{b^2 \cdot c}{5 \cdot \sqrt{3}}}\right)$	f)	$log_a\left(\sqrt[6]{\frac{\sqrt{2} \cdot \sqrt[4]{u}}{\sqrt[3]{v} \cdot w^{\frac{2}{3}}}}\right)$
g)	$log_a\left[\frac{3 \cdot \sqrt{\frac{1}{6}} \cdot c^{\frac{2}{3}}}{\left(\frac{d}{e}\right)^3}\right]^5$				

Aufgabe A2

Fasse nach den Logarithmusgesetzen zu einem Logarithmus zusammen.
(Alle Buchstaben vertreten positive Zahlen und es ist $a \neq 1$.)

a)	$\frac{1}{3} \cdot (2 \cdot log_a(b) - \frac{1}{2} log_a(c))$	b)	$log_a(3) + 2 \cdot log_a(p) - 3 \cdot log_a(q)$
c)	$log_a(a) + log_a(x) - 2 \cdot log_a(y)$	d)	$1 - log_a(x) - 2 \cdot log_a(y)$
e)	$\frac{3}{2} \cdot log_a(b) + \frac{2}{3} \cdot log_a(c) - 1$	f)	$\frac{1}{2} \cdot (log_a(x) - log_a(y))$
g)	$2 + 2 \cdot log_a(x) + \frac{1}{2} \cdot log_a(y) - 3 \cdot log_a(z)$		
h)	$4 \cdot log_a(u) - \left(\frac{1}{2} \cdot log_a(v) - 3 \cdot \log_a(w)\right)$		
i)	$\frac{1}{4} \cdot (2 + log_a(2) + 3 \cdot log_a(p) - 2 \cdot log_a(q) - 4 \cdot log_a(r))$		
j)	$b \cdot log_a(x) - \frac{1}{c} \cdot log_a(y) + d \cdot log_a(z) - \frac{e}{2}$		

Aufgabe A3

Zerlege den Logarithmusterm in Einzelterme.

a)	$log_a\left(\frac{3b + 2c}{b - 2d}\right)$	b)	$log_a((3x + 2y)(x - y))$
c)	$log_a((x + y)^3 \cdot (x - y)^5)$	d)	$log_a\left(\sqrt{(3u + 2v)^3}\right)$
e)	$log_a\left(x^{3+2y} \cdot \sqrt{z}\right)$	f)	$log_a\left(\sqrt{w} \cdot (u + v)^x\right)$
g)	$log_a\left(\frac{x \cdot y}{x + y}\right)$	h)	$log_a\left(\frac{u - v}{\sqrt{u^2 - v^2}} \cdot w^3\right)$
i)	$log_a\left(\frac{x \cdot (x - y)}{(3x + 2y)^2}\right)$	j)	$log_a\left(\sqrt{\frac{1 - x}{1 + x^2}}\right)$

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Lösung A1

a) $log_a\left(\sqrt[3]{\frac{2b^3c}{5de^5v}}\right) = log_a\left(\frac{b}{e}\cdot\sqrt[3]{\frac{2c}{5de^2v}}\right) = log_a\left(\frac{b}{e}\right) + \frac{1}{3}log_a\left(\frac{2c}{5de^2v}\right)$

$= log_a(b) - log_a(e) + \frac{1}{3}(log_a(2) + log_a(c) - (log_a(5) + log_a(d) + 2\cdot log_a(e) + log_a(v)))$

$= log_a(b) - \frac{5}{3}log_a(e) + \frac{1}{3}(log_a(2) + log_a(c) - log_a(5) - log_a(d) - log_a(v))$

b) $log_a\left(\frac{3b^2c^4\cdot\sqrt{d^2}}{\sqrt[3]{2e^2f^3}}\right) = log_a\left(\frac{3b^2c^4d}{f\cdot\sqrt[3]{2e^2}}\right) = \log_a\left(\frac{3b^2c^4d}{f}\cdot\frac{1}{\sqrt[3]{2e^2}}\right)$

$= log_a(3b^2c^4d) - log_a\left(f\cdot\sqrt[3]{2e^2}\right) = log_a(3) + 2\cdot log_a(b) + 4\cdot log_a(c) + log_a(d) - (log_a(f) + \frac{l}{3}\cdot log_a(2) + \frac{2}{3}log_a(e))$

c) $log_a\left(\frac{\sqrt[3]{e}\cdot b^2}{5\cdot\sqrt{3}}\right) = \frac{1}{3}\cdot log_a(e) + 2\cdot log_a(b) - (log_a(5) + \frac{1}{2}\cdot log_a(3))$

d) $log_a\left(\left(\frac{\sqrt[4]{u}\cdot 5^4}{v\cdot\sqrt[3]{12}}\right)^{\frac{1}{5}}\right) = \frac{1}{5}\cdot\left(log_a\left(\sqrt[4]{u}\cdot 5^4\right) - log_a\left(v\cdot\sqrt[3]{12}\right)\right) = \frac{1}{5}(\frac{1}{4}log_a(u) + 4\,log_a(5) - log_a(v) - \frac{1}{3}log_a(12))$

$= \frac{1}{12}log_a(u) + \frac{4}{5}log_a(5) - \frac{1}{5}log_a(v) - \frac{1}{15}log_a(12)$

e) $log_a\left(\sqrt[3]{\frac{b^2\cdot c}{5\cdot\sqrt{3}}}\right) = \frac{1}{3}\left(log_a(b^2\cdot c) - (log_a\left(5\cdot\sqrt{3}\right)\right)$

$= \frac{2}{3}log_a(b) + \frac{1}{3}log_a(c) - \frac{1}{3}log_a(5) - \frac{1}{6}log_a(3)$

f) $log_a\left(\sqrt[6]{\frac{\sqrt{2}\cdot\sqrt[4]{u}}{\sqrt[3]{v}\cdot w^{\frac{2}{3}}}}\right) = \frac{1}{6}\cdot\left(\log_a\left(\sqrt{2}\cdot\sqrt[4]{u}\right) - log_a\left(\sqrt[3]{v}\cdot w^{\frac{2}{3}}\right)\right)$

$= \frac{1}{12}log_a(2) + \frac{1}{24}log_a(u) - \frac{1}{18}log_a(v) - \frac{1}{9}log_a(w)$

g) $log_a\left[\frac{3\cdot\sqrt{\frac{1}{6}}\cdot c^{\frac{2}{3}}}{\left(\frac{d}{e}\right)^3}\right]^5 = 5\cdot\left(log_a\left(3\cdot\sqrt{\frac{1}{6}}\cdot c^{\frac{2}{3}}\right) - log_a\left(\left(\frac{d}{e}\right)^3\right)\right)$

$= 5\cdot(log_a(3) + \frac{1}{2}\cdot log_a\left(\frac{1}{6}\right) + \frac{2}{3}\cdot log_a(c) - 3\cdot(log_a(d) - log_a(e)))$

$= 5\cdot(log_a(3) - \frac{1}{2}\cdot log_a(6) + \frac{2}{3}\cdot log_a(c) - 3\cdot log_a(d) + 3\cdot log_a(e))$

Lösung A2

a) $\frac{1}{3}\cdot(2\cdot log_a(b) - \frac{1}{2}log_a(c)) = log_a\left(\sqrt[3]{\frac{b^2}{\sqrt{c}}}\right)$

b) $log_a(3) + 2\cdot log_a(p) - 3\cdot log_a(q) = log_a\left(\frac{3\cdot p^2}{q^3}\right)$

c) $log_a(a) + log_a(x) - 2\cdot log_a(y) = log_a\left(\frac{a\cdot x}{y^2}\right)$

d) $1 - log_a(x) - 2\cdot log_a(y) = log_a(a) - log_a(x) - 2\cdot log_a(y) = log_a\left(\frac{a}{x\cdot y^2}\right)$

e) $\frac{3}{2}\cdot log_a(b) + \frac{2}{3}\cdot log_a(c) - 1 = \frac{3}{2}\cdot log_a(b) + \frac{2}{3}\cdot log_a(c) - log_a(a) = log_a\left(\frac{\sqrt{b^3}\cdot\sqrt[3]{c^2}}{a}\right)$

f) $\frac{1}{2}\cdot(log_a(x) - log_a(y)) = \log_a\left(\sqrt{\frac{x}{y}}\right)$

g) $2 + 2\cdot log_a(x) + \frac{1}{2}\cdot log_a(y) - 3\cdot log_a(z) = 2log_a(a) + 2log_a(x) + \frac{1}{2}log_a(y) - 3\,log_a(z)$

$= log_a\left(\frac{a^2\cdot x^2\cdot\sqrt{y}}{z^3}\right)$

h) $4 \cdot log_a(u) - \left(\frac{1}{2} \cdot log_a(v) - 3 \cdot \log_a(w)\right) = 4 \cdot log_a(u) - \frac{1}{2} \cdot log_a(v) + 3 \cdot \log_a(w)$
$= log_a\left(\frac{u^4 \cdot w^3}{\sqrt{v}}\right)$

i) $\frac{1}{4} \cdot (2 + log_a(2) + 3 \cdot log_a(p) - 2 \cdot log_a(q) - 4 \cdot log_a(r)\ ) = log_a\left(\sqrt[4]{\frac{2a^2 \cdot p^3}{q^2 \cdot r^4}}\right)$

Lösung A3

a) $log_a\left(\frac{3b+2c}{b-2d}\right) = log_a(3b + 2c) - log_a(b - 2d)$

b) $log_a((3x + 2y)(x - y)) \ = log_a(3x + 2y) + log_a(x - y)$

c) $log_a((x + y)^3 \cdot (x - y)^5) = 3 \cdot log_a(x + y) + 5 \cdot log_a(x - y)$

d) $log_a\left(\sqrt{(3u + 2v)^3}\right) = \frac{3}{2} \cdot log_a(3u + 2v)$

e) $log_a\left(x^{3+2y} \cdot \sqrt{z}\right) = (3 + 2y) \cdot log_a(x) + \frac{1}{2} \cdot log_a(z)$

f) $log_a\left(\sqrt{w} \cdot (u + v)^x\right) = \frac{1}{2} \cdot log_a(w) + x \cdot log_a(u + v)$

g) $log_a\left(\frac{x \cdot y}{x+y}\right) = log_a(x) + log_a(y) - log_a(x + y)$

h) $log_a\left(\frac{u-v}{\sqrt{u^2-v^2}} \cdot w^3\right) = log_a(u - v) + 3 \cdot log_a(w) - \frac{1}{2} \cdot log_a(u^2 - v^2)$

i) $log_a\left(\frac{x \cdot (x-y)}{(3x+2y)^2}\right) = log_a(x) + log_a(x - y) - 2 \cdot log_a(3x + 2y)$

j) $log_a\left(\sqrt{\frac{1-x}{1+x^2}}\right) = \frac{1}{2} \cdot (log_a(1 - x) - log_a(1 + x^2))$