

Lösung A1

- a)
$$\log_a \left(\sqrt[3]{\frac{2b^3c}{5de^5v}} \right) = \log_a \left(\frac{b}{e} \cdot \sqrt[3]{\frac{2c}{5de^2v}} \right) = \log_a \left(\frac{b}{e} \right) + \frac{1}{3} \log_a \left(\frac{2c}{5de^2v} \right)$$

$$= \log_a(b) - \log_a(e) + \frac{1}{3} (\log_a(2) + \log_a(c) - (\log_a(5) + \log_a(d) + 2 \cdot \log_a(e) + \log_a(v)))$$

$$= \log_a(b) - \frac{5}{3} \log_a(e) + \frac{1}{3} (\log_a(2) + \log_a(c) - \log_a(5) - \log_a(d) - \log_a(v))$$
- b)
$$\log_a \left(\frac{3b^2c^4 \cdot \sqrt{d^2}}{\sqrt[3]{2e^2f^3}} \right) = \log_a \left(\frac{3b^2c^4d}{f \cdot \sqrt[3]{2e^2}} \right) = \log_a \left(\frac{3b^2c^4d}{f} \cdot \frac{1}{\sqrt[3]{2e^2}} \right)$$

$$= \log_a(3b^2c^4d) - \log_a(f \cdot \sqrt[3]{2e^2}) = \log_a(3) + 2 \cdot \log_a(b) + 4 \cdot \log_a(c) + \log_a(d) - (\log_a(f) + \frac{1}{3} \cdot \log_a(2) + \frac{2}{3} \log_a(e))$$
- c)
$$\log_a \left(\frac{\sqrt[3]{e} \cdot b^2}{5 \cdot \sqrt{3}} \right) = \frac{1}{3} \cdot \log_a(e) + 2 \cdot \log_a(b) - (\log_a(5) + \frac{1}{2} \cdot \log_a(3))$$
- d)
$$\log_a \left(\left(\frac{\sqrt[4]{u} \cdot 5^4}{v \cdot \sqrt[3]{12}} \right)^{\frac{1}{5}} \right) = \frac{1}{5} \cdot (\log_a(\sqrt[4]{u} \cdot 5^4) - \log_a(v \cdot \sqrt[3]{12})) = \frac{1}{5} \left(\frac{1}{4} \log_a(u) + 4 \log_a(5) - \log_a(v) - \frac{1}{3} \log_a(12) \right)$$

$$= \frac{1}{12} \log_a(u) + \frac{4}{5} \log_a(5) - \frac{1}{5} \log_a(v) - \frac{1}{15} \log_a(12)$$
- e)
$$\log_a \left(\sqrt[3]{\frac{b^2 \cdot c}{5 \cdot \sqrt{3}}} \right) = \frac{1}{3} (\log_a(b^2 \cdot c) - (\log_a(5 \cdot \sqrt{3})))$$

$$= \frac{2}{3} \log_a(b) + \frac{1}{3} \log_a(c) - \frac{1}{3} \log_a(5) - \frac{1}{6} \log_a(3)$$
- f)
$$\log_a \left(\sqrt[6]{\frac{\sqrt{2} \cdot \sqrt[4]{u}}{\sqrt[3]{v} \cdot w^{\frac{2}{3}}}} \right) = \frac{1}{6} \cdot (\log_a(\sqrt{2} \cdot \sqrt[4]{u}) - \log_a(\sqrt[3]{v} \cdot w^{\frac{2}{3}}))$$

$$= \frac{1}{12} \log_a(2) + \frac{1}{24} \log_a(u) - \frac{1}{18} \log_a(v) - \frac{1}{9} \log_a(w)$$
- g)
$$\log_a \left[\frac{3 \cdot \sqrt[6]{\frac{1}{c^3}}}{\left(\frac{d}{e}\right)^3} \right]^5 = 5 \cdot \left(\log_a \left(3 \cdot \sqrt[6]{\frac{1}{c^3}} \right) - \log_a \left(\left(\frac{d}{e}\right)^3 \right) \right)$$

$$= 5 \cdot \left(\log_a(3) + \frac{1}{2} \cdot \log_a \left(\frac{1}{c} \right) + \frac{2}{3} \cdot \log_a(c) - 3 \cdot (\log_a(d) - \log_a(e)) \right)$$

$$= 5 \cdot \left(\log_a(3) - \frac{1}{2} \cdot \log_a(c) + \frac{2}{3} \cdot \log_a(c) - 3 \cdot \log_a(d) + 3 \cdot \log_a(e) \right)$$

Lösung A2

- a)
$$\frac{1}{3} \cdot (2 \cdot \log_a(b) - \frac{1}{2} \log_a(c)) = \log_a \left(\sqrt[3]{\frac{b^2}{\sqrt{c}}} \right)$$
- b)
$$\log_a(3) + 2 \cdot \log_a(p) - 3 \cdot \log_a(q) = \log_a \left(\frac{3 \cdot p^2}{q^3} \right)$$
- c)
$$\log_a(a) + \log_a(x) - 2 \cdot \log_a(y) = \log_a \left(\frac{a \cdot x}{y^2} \right)$$
- d)
$$1 - \log_a(x) - 2 \cdot \log_a(y) = \log_a(a) - \log_a(x) - 2 \cdot \log_a(y) = \log_a \left(\frac{a}{x \cdot y^2} \right)$$
- e)
$$\frac{3}{2} \cdot \log_a(b) + \frac{2}{3} \cdot \log_a(c) - 1 = \frac{3}{2} \cdot \log_a(b) + \frac{2}{3} \cdot \log_a(c) - \log_a(a) = \log_a \left(\frac{\sqrt{b^3} \cdot \sqrt[3]{c^2}}{a} \right)$$
- f)
$$\frac{1}{2} \cdot (\log_a(x) - \log_a(y)) = \log_a \left(\sqrt{\frac{x}{y}} \right)$$
- g)
$$2 + 2 \cdot \log_a(x) + \frac{1}{2} \cdot \log_a(y) - 3 \cdot \log_a(z) = 2 \log_a(a) + 2 \log_a(x) + \frac{1}{2} \log_a(y) - 3 \log_a(z)$$

$$= \log_a \left(\frac{a^2 \cdot x^2 \cdot \sqrt{y}}{z^3} \right)$$

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$$h) \quad 4 \cdot \log_a(u) - \left(\frac{1}{2} \cdot \log_a(v) - 3 \cdot \log_a(w) \right) = 4 \cdot \log_a(u) - \frac{1}{2} \cdot \log_a(v) + 3 \cdot \log_a(w)$$

$$= \log_a \left(\frac{u^4 \cdot w^3}{\sqrt{v}} \right)$$

$$i) \quad \frac{1}{4} \cdot (2 + \log_a(2) + 3 \cdot \log_a(p) - 2 \cdot \log_a(q) - 4 \cdot \log_a(r)) = \log_a \left(\sqrt[4]{\frac{2a^2 \cdot p^3}{q^2 \cdot r^4}} \right)$$

Lösung A3

$$a) \quad \log_a \left(\frac{3b+2c}{b-2d} \right) = \log_a(3b+2c) - \log_a(b-2d)$$

$$b) \quad \log_a((3x+2y)(x-y)) = \log_a(3x+2y) + \log_a(x-y)$$

$$c) \quad \log_a((x+y)^3 \cdot (x-y)^5) = 3 \cdot \log_a(x+y) + 5 \cdot \log_a(x-y)$$

$$d) \quad \log_a \left(\sqrt{(3u+2v)^3} \right) = \frac{3}{2} \cdot \log_a(3u+2v)$$

$$e) \quad \log_a(x^{3+2y} \cdot \sqrt{z}) = (3+2y) \cdot \log_a(x) + \frac{1}{2} \cdot \log_a(z)$$

$$f) \quad \log_a(\sqrt{w} \cdot (u+v)^x) = \frac{1}{2} \cdot \log_a(w) + x \cdot \log_a(u+v)$$

$$g) \quad \log_a \left(\frac{x \cdot y}{x+y} \right) = \log_a(x) + \log_a(y) - \log_a(x+y)$$

$$h) \quad \log_a \left(\frac{u-v}{\sqrt{u^2-v^2}} \cdot w^3 \right) = \log_a(u-v) + 3 \cdot \log_a(w) - \frac{1}{2} \cdot \log_a(u^2-v^2)$$

$$i) \quad \log_a \left(\frac{x \cdot (x-y)}{(3x+2y)^2} \right) = \log_a(x) + \log_a(x-y) - 2 \cdot \log_a(3x+2y)$$

$$j) \quad \log_a \left(\sqrt{\frac{1-x}{1+x^2}} \right) = \frac{1}{2} \cdot (\log_a(1-x) - \log_a(1+x^2))$$