Aufgabenblatt
zu Potenzen mit rationalem Exponenten
Lösungen

Level 2 – Fortgeschritten – Blatt 2

Lösung A1

a) $\frac{8x^{\frac{1}{4}}}{4^{\frac{1}{4}}} = \frac{2^3 \cdot x^{\frac{1}{4}}}{2^{2^{\frac{1}{4}}}} = \frac{2^3 \cdot x^{\frac{1}{4}}}{2^{\frac{1}{2}}} = 2^{\frac{5}{2}} \cdot x^{\frac{1}{4}}$

b) $3^{\frac{1}{t}+\frac{1}{x+1}+\frac{1}{2t}+\frac{1}{x}} = 3^{\frac{2x(x+1)+2tx+x(x+1)+2t(x+1)}{2tx(x+1)}} = 3^{\frac{2x^2+2x+2tx+x^2+x+2tx+2t}{2tx(x+1)}} = 3^{\frac{3x^2+3x+4tx+2t}{2tx(x+1)}}$

c) $3^{3+\frac{1}{x-2}} = 3^{\frac{3(x-2)+1}{x-2}} = 3^{\frac{3x-5}{x-2}}$

d) $(e-3)^{\frac{5}{4}}$

e) $e^{\frac{1}{2x}+\frac{1}{a-x}-\frac{1}{x+a}} = e^{\frac{(a+x)(a-x)+2x(a+x)-2x(a-x)}{2x(a+x)(a-x)}} = e^{\frac{a^2-x^2+2ax+2x^2-2ax+2x^2}{2x(a^2-x^2)}} = e^{\frac{a^2+3x^2}{2x(a^2-x^2)}}$

f) $\frac{9a^{\frac{1}{2}}b}{3ab^{\frac{1}{2}}} = \frac{3b^{\frac{1}{2}}}{a^{\frac{1}{2}}} = 3 \cdot \left(\frac{b}{a}\right)^{\frac{1}{2}}$

g) $\frac{14a^{\frac{1}{2}}b^{\frac{1}{3}}}{7a^{\frac{1}{3}}b^{\frac{1}{2}}x} = \frac{2a^{\frac{1}{6}}}{b^{\frac{1}{6}}x} = \frac{2}{x} \cdot \left(\frac{a}{b}\right)^{\frac{1}{6}}$

h) $\frac{8a^{\frac{1}{2}}x}{2ax^{\frac{1}{3}}} = \frac{4x^{\frac{2}{3}}}{a^{\frac{1}{2}}}$

i) $\frac{6a^{\frac{1}{3}}x^{\frac{1}{3}}y^0}{3a^{\frac{1}{2}}y^{\frac{1}{2}}} = \frac{2x^{\frac{1}{3}}}{a^{\frac{1}{6}}y^{\frac{1}{2}}}$

j) $\frac{12a^{\frac{1}{3}}xy}{4a^{\frac{1}{2}}y^{\frac{1}{3}}} = \frac{3xy^{\frac{2}{3}}}{a^{\frac{1}{6}}}$

k) $\frac{6a^{\frac{1}{2}}b^{\frac{1}{2}}x}{3ab^{\frac{1}{3}}x^{\frac{1}{3}}} = \frac{2b^{\frac{1}{6}}x^{\frac{2}{3}}}{a^{\frac{1}{2}}}$

l) $\frac{10a^{\frac{1}{2}}x^{\frac{1}{3}}y}{2ax^{\frac{1}{2}}y^{\frac{1}{2}}} = \frac{5y^{\frac{1}{2}}}{a^{\frac{1}{2}}x^{\frac{1}{6}}}$

m) $\frac{16a^{\frac{1}{2}}bx^{\frac{1}{2}}}{4ab^{\frac{1}{3}}x^{\frac{1}{3}}} = \frac{4b^{\frac{2}{3}}x^{\frac{1}{6}}}{a^{\frac{1}{2}}}$

n) $\frac{a^{\frac{1}{4}}b^{\frac{1}{n+3}}}{a^{\frac{1}{n}}b^{\frac{1}{2n-1}}} = a^{\frac{1}{4}-\frac{1}{n}}b^{\frac{1}{n+3}-\frac{1}{(2n-1)}} = a^{\frac{n-4}{4n}}b^{\frac{2n-1-n-2}{(n+2)(2n-1)}} = a^{\frac{n-4}{4n}}b^{\frac{n-3}{(n+2)(2n-1)}}$

o) $\frac{4^{\frac{1}{x+2}}}{16} = 4^{\frac{1}{x+2}} \cdot 4^{-2} = 4^{\frac{1}{x+2}-2} = 4^{\frac{1-2x-4}{x+2}} = 4^{-\frac{2x+3}{x+2}} = \frac{1}{4^{\frac{2x+3}{x+2}}}$

p) $\frac{81}{3^{\frac{1}{x+3}}} = 3^{4-\frac{1}{x+3}} = 3^{\frac{4x+11}{x+3}}$

q) $(a-b)^{\frac{1}{3}-\frac{1}{(n-1)}} = (a-b)^{\frac{n-1-3}{3\cdot(n-1)}} = (a-b)^{\frac{n-4}{3(n-1)}}$

r) $\frac{a^{\frac{1}{n+1}}}{a^{\frac{1}{n}}} = a^{\frac{1}{n+1}-\frac{1}{n}} = a^{\frac{n-n-1}{n(n+1)}} = a^{-\frac{1}{n(n+1)}} = \frac{1}{a^{\frac{1}{n(n+1)}}}$

Lösung A2

a) $3^{\frac{1}{2n+2}}$

b) $16x^2 + 24xy^{\frac{1}{2}} + 9y$

c) $-x^{\frac{1}{2}} + 4x^{\frac{1}{4}} - 4$

d) $x - x^{\frac{2}{3}}$

e) $9x + 12x^{\frac{1}{2}}t + 4t^2$

f) $3^{\frac{1}{2}}\left(x - 2x^{\frac{1}{2}}t + t^2\right)$

Lösung A3

a) $4a^{\frac{1}{3}}xb^{-1}$

b) $4a^{\frac{1}{2}}yb^{-\frac{1}{3}}$

c) $2a^{-\frac{1}{2}}yb^{-\frac{1}{2}} = 2y(ab)^{-\frac{1}{2}}$

d) $2a^{\frac{2}{3}}b^{\frac{1}{2}}x^{\frac{1}{2}} = 2a^{\frac{2}{3}}(bx)^{\frac{1}{2}}$

e) $15a^{\frac{1}{3}}b^{-\frac{1}{2}}xy^{-1}$

f) $3a^{-\frac{2}{3}}x^{\frac{1}{2}}b^{-\frac{1}{2}}$

g) $5a^{\frac{1}{2}}x^{-\frac{2}{3}}$

h) $4a^{-\frac{1}{2}}b^{-\frac{1}{6}}c^{-1}$

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Lösung A4

a) $3\left(a^{\frac{1}{2}}+2a^{\frac{1}{3}}\right)$

b) $2(a^{\frac{1}{2}}-3a^{\frac{1}{3}}+2a^{\frac{1}{4}}-4a^{\frac{1}{5}})$

c) $3(x-2)\left(\frac{1}{2}x^{\frac{1}{2}}-1\right)^2$

d) $\frac{1}{2}\left(e^{\frac{1}{x}}-\frac{1}{2}e^{\frac{1}{x+1}}\right)$

e) $3\left(x^{\frac{1}{4}}-4x^{\frac{1}{2}}\right)$

f) keine Vereinfachung möglich

g) $a^{\frac{1}{5b}}+3a^{\frac{5}{5b}}=a^{\frac{1}{5b}}\left(1+3a^{\frac{4}{5b}}\right)$

h) keine Vereinfachung möglich

i) keine Vereinfachung möglich

Lösung A5

a) $x^{\frac{1}{2}}\left(x^{\frac{1}{2}}+2\right)$

b) $3a^{\frac{1}{3}}\left(a^{\frac{1}{3}}-4\right)$

c) $(x^{\frac{1}{8}}+a^{\frac{1}{4}})(x^{\frac{1}{8}}-a^{\frac{1}{4}})$

d) $(e^{\frac{1}{2x}}+e^{\frac{1}{6x}})(e^{\frac{1}{2x}}-e^{\frac{1}{6x}})$

e) $(e^{\frac{1}{4x}}+1)(e^{\frac{1}{4x}}-1)$

f) $e^{\frac{1}{x}}(x+1)^2$