

Lösung A1

a) $\frac{8}{\sqrt{2}} = \frac{8 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{8 \cdot \sqrt{2}}{2} = 4 \cdot \sqrt{2}$
 c) $\frac{\sqrt{14}}{3 \cdot \sqrt{5}} = \frac{\sqrt{14} \cdot \sqrt{5}}{3 \cdot \sqrt{5} \cdot \sqrt{5}} = \frac{\sqrt{70}}{15} = \frac{1}{15} \sqrt{70}$
 e) $\sqrt{\frac{3}{5}} = \frac{\sqrt{3} \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{\sqrt{15}}{5} = \frac{1}{5} \cdot \sqrt{15}$
 g) $\frac{5 \cdot \sqrt{3}}{3 \cdot \sqrt{5}} = \frac{5 \sqrt{3} \cdot \sqrt{5}}{3 \sqrt{5} \cdot \sqrt{5}} = \frac{5 \sqrt{15}}{15} = \frac{1}{3} \cdot \sqrt{15}$
 i) $\frac{\sqrt{3}-2}{\sqrt{5}} = \frac{(\sqrt{3}-2) \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{\sqrt{15}-2\sqrt{5}}{5}$
 k) $\frac{3\sqrt{27}-2\sqrt{48}}{2\sqrt{3}} = \frac{(3\sqrt{27}-2\sqrt{48}) \cdot \sqrt{3}}{2\sqrt{3} \cdot \sqrt{3}} = \frac{27-24}{6} = \frac{1}{2}$

b) $\frac{3}{5 \cdot \sqrt{3}} = \frac{3 \cdot \sqrt{3}}{5 \cdot \sqrt{3} \cdot \sqrt{3}} = \frac{3 \cdot \sqrt{3}}{15} = \frac{1}{5} \cdot \sqrt{3}$
 d) $\frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{6}}{3} = \frac{1}{3} \cdot \sqrt{6}$
 f) $\sqrt{5 \frac{2}{3}} = \frac{\sqrt{17} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{51}}{3} = \frac{1}{3} \cdot \sqrt{51}$
 h) $\frac{7 \cdot \sqrt{14}}{3 \cdot \sqrt{35}} = \frac{7 \sqrt{14} \cdot \sqrt{35}}{3 \sqrt{35} \cdot \sqrt{35}} = \frac{7 \sqrt{490}}{105} = \frac{7 \cdot 7 \sqrt{10}}{15 \cdot 7} = \frac{7}{15} \sqrt{10}$
 j) $\frac{\sqrt{12}-\sqrt{7}}{\sqrt{3}} = \frac{(\sqrt{12}-\sqrt{7}) \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{60}-\sqrt{35}}{3}$
 l) $\frac{4\sqrt{5}+3\sqrt{8}}{5\sqrt{20}} = \frac{(4\sqrt{5}+3\sqrt{8}) \cdot \sqrt{20}}{5\sqrt{20} \cdot \sqrt{20}} = \frac{40-12\sqrt{10}}{100} = \frac{20-6\sqrt{10}}{50}$

Lösung A2

a) $\frac{\sqrt{5}}{\sqrt{5}-\sqrt{3}} = \frac{\sqrt{5} \cdot (\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} = \frac{5+\sqrt{15}}{2}$
 c) $\frac{5\sqrt{2}+2\sqrt{5}}{3} = \frac{(5\sqrt{2}+2\sqrt{5})(5\sqrt{2}-2\sqrt{5})}{3(5\sqrt{2}-2\sqrt{5})} = \frac{15\sqrt{2}-6\sqrt{5}}{3(5\sqrt{2}-2\sqrt{5})} = \frac{50-20}{30} = \frac{30}{30} = 1$
 e) $\frac{8\sqrt{7}}{8\sqrt{7}-7\sqrt{8}} = \frac{8\sqrt{7} \cdot (8\sqrt{7}+7\sqrt{8})}{(8\sqrt{7}-7\sqrt{8})(8\sqrt{7}+7\sqrt{8})} = \frac{448+56\sqrt{56}}{448-392} = \frac{448-112\sqrt{14}}{56} = 8 - 2\sqrt{14}$
 g) $\frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = 7 + 4\sqrt{3}$
 i) $\frac{5\sqrt{3}+3\sqrt{5}}{5\sqrt{3}-3\sqrt{5}} = \frac{(5\sqrt{3}+3\sqrt{5})(5\sqrt{3}+3\sqrt{5})}{(5\sqrt{3}-3\sqrt{5})(5\sqrt{3}+3\sqrt{5})} = \frac{75+30\sqrt{15}+45}{75-45} = 4 + \sqrt{15}$
 k) $\frac{3\sqrt{3}-2}{2\sqrt{2}+3} = \frac{(3\sqrt{3}-2)(2\sqrt{2}-3)}{(2\sqrt{2}+3)(2\sqrt{2}-3)} = \frac{-(3\sqrt{3}-2) \cdot (2\sqrt{2}-3)}{2-9} = \frac{-(3\sqrt{3}-2) \cdot (2\sqrt{2}-3)}{-7} = \frac{(3\sqrt{3}-2) \cdot (2\sqrt{2}-3)}{7}$

b) $\frac{7}{\sqrt{10}-\sqrt{5}} = \frac{7 \cdot (\sqrt{10}+\sqrt{5})}{(\sqrt{10}-\sqrt{5})(\sqrt{10}+\sqrt{5})} = \frac{7 \cdot (\sqrt{10}+\sqrt{5})}{5}$
 d) $\frac{\sqrt{12}}{3\sqrt{3}+4\sqrt{12}} = \frac{\sqrt{12} \cdot (3\sqrt{3}-4\sqrt{12})}{(3\sqrt{3}+4\sqrt{12})(3\sqrt{3}-4\sqrt{12})} = \frac{18-48}{27-192} = \frac{2}{11}$
 f) $\frac{2-\sqrt{3}}{2+\sqrt{3}} = \frac{(2-\sqrt{3})(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{4-4\sqrt{3}+3}{4-3} = 7 - 4\sqrt{3}$
 h) $\frac{\sqrt{15}-\sqrt{5}}{\sqrt{6}-\sqrt{2}} = \frac{(\sqrt{15}-\sqrt{5})(\sqrt{6}+\sqrt{2})}{(\sqrt{6}-\sqrt{2})(\sqrt{6}+\sqrt{2})} = \frac{\sqrt{90}-\sqrt{10}}{6-2} = \frac{1}{2} \sqrt{10}$
 j) $\frac{\sqrt{10}-\sqrt{2}}{\sqrt{5}+\sqrt{2}} = \frac{(\sqrt{10}-\sqrt{2})(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})} = \frac{\sqrt{50}-\sqrt{20}-\sqrt{10}+2}{3}$
 l) $\frac{2\sqrt{3}-3\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{(2\sqrt{3}-3\sqrt{2})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})} = -\sqrt{6}$

Lösung A30

a) $\sqrt{\frac{a}{a+b}} = \frac{\sqrt{a} \cdot \sqrt{a+b}}{\sqrt{a+b} \cdot \sqrt{a+b}} = \frac{\sqrt{a^2+ab}}{a+b}$
 $[a \geq 0; a+b > 0]$
 c) $\sqrt{\frac{a+b}{a}} = \frac{\sqrt{a} \cdot \sqrt{a+b}}{\sqrt{a} \cdot \sqrt{a}} = \frac{\sqrt{a^2+ab}}{a}$
 $[a+b \geq 0; a > 0]$

b) $\frac{3\sqrt{15}+5\sqrt{125}-2\sqrt{20}}{12\sqrt{5}} = \frac{(3\sqrt{15}+5\sqrt{125}-2\sqrt{20}) \cdot \sqrt{5}}{12\sqrt{5} \cdot \sqrt{5}} = \frac{15\sqrt{3}+125-20}{60} = \frac{15\sqrt{3}+105}{60} = \frac{\sqrt{3}+7}{4}$
 d) $\frac{3\sqrt{a}}{\sqrt{2a+3b}} = \frac{3\sqrt{a} \cdot \sqrt{2a+3b}}{\sqrt{2a+3b} \cdot \sqrt{2a+3b}} = \frac{3\sqrt{2a^2+3ab}}{2a+3b}$
 $[a \geq 0; 2a+3b > 0]$